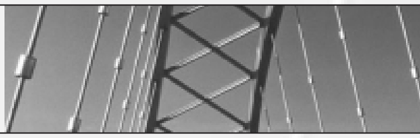


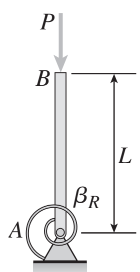
11

Columns

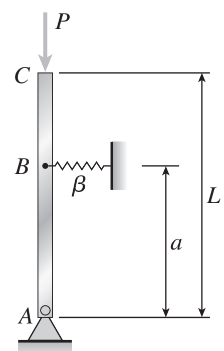
Idealized Buckling Models

Problem 11.2-1 through 11.2-4 The figure shows an idealized structure consisting of one or more **rigid bars** with pinned connections and linearly elastic springs. Rotational stiffness is denoted β_R and translational stiffness is denoted β.

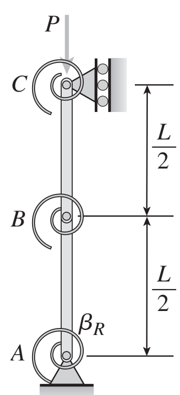
Determine the critical load P_{cr} for the structure.



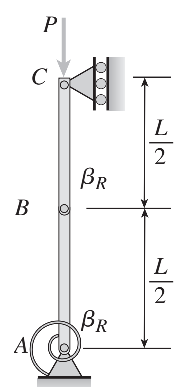
Prob. 11.2-1



Prob. 11.2-2

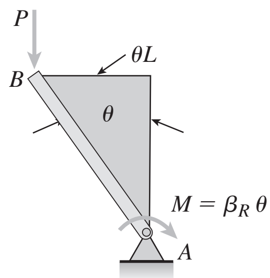


Prob. 11.2-3



Prob. 11.2-4

Solution 11.2-1 Rigid bar AB

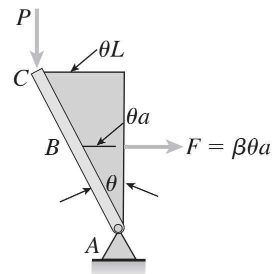


$$\sum M_A = 0$$

$$P(\theta L) - \beta_R \theta = 0$$

$$P_{cr} = \frac{\beta_R}{L} \quad \longleftarrow$$

Solution 11.2-2 Rigid bar *ABC*

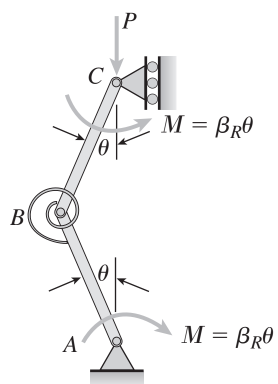


$$\sum M_A = 0$$

$$P\theta L - \beta\theta a^2 = 0$$

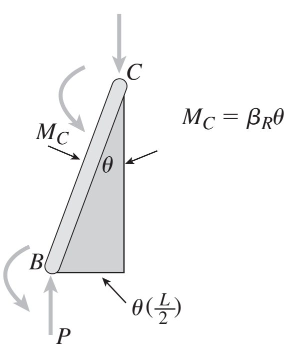
$$P_{cr} = \frac{\beta a^2}{L} \quad \longleftarrow$$

Solution 11.2-3 Two rigid bars with a pin connection



$\sum M_A = 0$ Shows that there are no horizontal reactions at the supports.

FREE-BODY DIAGRAM OF BAR *BC*



$$M_C = \beta_R\theta$$

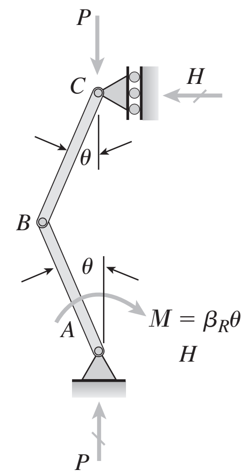
$$M_B = \beta_R(2\theta)$$

$$\sum M_B = 0 \quad M_B + M_C - P\theta\left(\frac{L}{2}\right) = 0$$

$$\beta_R(2\theta) + \beta_R\theta = \frac{PL\theta}{2}$$

$$P_{cr} = \frac{6\beta_R}{L} \quad \longleftarrow$$

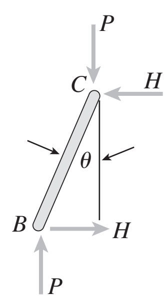
Solution 11.2-4 Two rigid bars with a pin connection



$$\sum M_A = 0 \quad HL - \beta_R\theta = 0$$

$$H = \frac{\beta_R\theta}{L}$$

FREE-BODY DIAGRAM OF BAR *BC*



$$\sum M_B = 0 \quad H\left(\frac{L}{2}\right) - P\left(\frac{\theta L}{2}\right) = 0$$

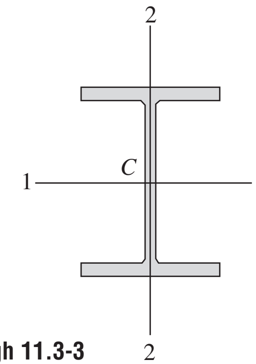
$$P_{cr} = \frac{H}{\theta} = \frac{\beta_R}{L} \quad \longleftarrow$$

Critical Loads of Columns with Pinned Supports

The problems for Section 11.3 are to be solved using the assumptions of ideal, slender, prismatic, linearly elastic columns (Euler buckling). Buckling occurs in the plane of the figure unless stated otherwise.

Problem 11.3-1 Calculate the critical load P_{cr} for a W 8 × 35 steel column (see figure) having length $L = 24$ ft and $E = 30 \times 10^6$ psi under the following conditions:

(a) The column buckles by bending about its strong axis (axis 1-1), and (b) the column buckles by bending about its weak axis (axis 2-2). In both cases, assume that the column has pinned ends.



Probs. 11.3-1 through 11.3-3

Solution 11.3-1 Column with pinned supports

W 8 × 35 steel column

$L = 24 \text{ ft} = 288 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$

$I_1 = 127 \text{ in.}^4 \quad I_2 = 42.6 \text{ in.}^4 \quad A = 10.3 \text{ in.}^2$

(a) Buckling about strong axis

$$P_{cr} = \frac{\pi^2 E I_1}{L^2} = 453 \text{ k} \quad \longleftarrow$$

(b) Buckling about weak axis

$$P_{cr} = \frac{\pi^2 E I_2}{L^2} = 152 \text{ k} \quad \longleftarrow$$

Note: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{453 \text{ k}}{10.3 \text{ in.}^2} = 44 \text{ ksi}$

$\therefore$ Solution is satisfactory if $\sigma_{PL} \geq 44$ ksi

---

Problem 11.3-2 Solve the preceding problem for a W 10 × 60 steel column having length $L = 30$ ft.

Solution 11.3-2 Column with pinned supports

W 10 × 60 steel column

$L = 30 \text{ ft} = 360 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$

$I_1 = 341 \text{ in.}^4 \quad I_2 = 116 \text{ in.}^4 \quad A = 17.6 \text{ in.}^2$

(a) Buckling about strong axis

$$P_{cr} = \frac{\pi^2 E I_1}{L^2} = 779 \text{ k} \quad \longleftarrow$$

(b) Buckling about weak axis

$$P_{cr} = \frac{\pi^2 E I_2}{L^2} = 265 \text{ k} \quad \longleftarrow$$

Note: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{779 \text{ k}}{17.6 \text{ in.}^2} = 44 \text{ ksi}$

$\therefore$ Solution is satisfactory if $\sigma_{PL} \geq 44$ ksi

---

Problem 11.3-3 Solve Problem 11.3-1 for a W 10 × 45 steel column having length $L = 28$ ft.

Solution 11.3-3 Column with pinned supports

W 10 × 45 steel column

$L = 28 \text{ ft} = 336 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$

$I_1 = 248 \text{ in.}^4 \quad I_2 = 53.4 \text{ in.}^4 \quad A = 13.3 \text{ in.}^2$

(a) Buckling about strong axis

$$P_{cr} = \frac{\pi^2 E I_1}{L^2} = 650 \text{ k} \quad \longleftarrow$$

(b) Buckling about weak axis

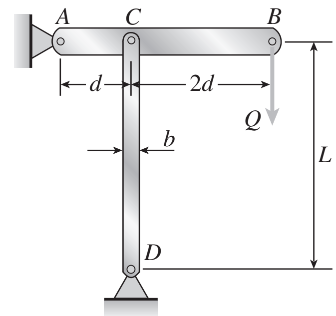
$$P_{cr} = \frac{\pi^2 E I_2}{L^2} = 140 \text{ k} \quad \longleftarrow$$

Note: $\sigma_{cr} = \dfrac{P_{CR}}{A} = \dfrac{650 \text{ k}}{13.3 \text{ in.}^2} = 49 \text{ ksi}$

$\therefore$ Solution is satisfactory if $\sigma_{PL} \geq 49$ ksi

Problem 11.3-4 A horizontal beam AB is pin-supported at end A and carries a load Q at end B, as shown in the figure. The beam is supported at C by a pinned-end column. The column is a solid steel bar ($E = 200$ GPa) of square cross section having length $L = 1.8$ m and side dimensions $b = 60$ mm.

Based upon the critical load of the column, determine the allowable load Q if the factor of safety with respect to buckling is $n = 2.0$.



Probs. 11.3-4 and 11.3-5

Solution 11.3-4 Beam supported by a column

COLUMN CD (STEEL)

$E = 200$ GPa $\quad L = 1.8$ m

Square cross section: $b = 60$ mm

Factor of safety: $n = 2.0$

$$I = \frac{b^4}{12} = 1.08 \times 10^6\,\text{mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 657.97\,\text{kN}$$

BEAM ACB $\quad \sum M_A = 0 \quad Q = \frac{P}{3}$

$$Q_{allow} = \frac{P_{allow}}{3} = \frac{P_{cr}}{3n} = \frac{P_{cr}}{6.0} = 109.7\,\text{kN} \longleftarrow$$

Problem 11.3-5 Solve the preceding problem if the column is aluminum ($E = 10 \times 10^6$ psi), the length $L = 30$ in., the side dimension $b = 1.5$ in., and the factor of safety $n = 1.8$.

Solution 11.3-5 Beam supported by a column

COLUMN CD (STEEL)

$E = 10 \times 10^6$ psi $\quad L = 30$ in.

Square cross section: $b = 1.5$ in.

Factor of safety: $n = 1.8$

$$I = \frac{b^4}{12} = 0.42188\,\text{in.}^4$$

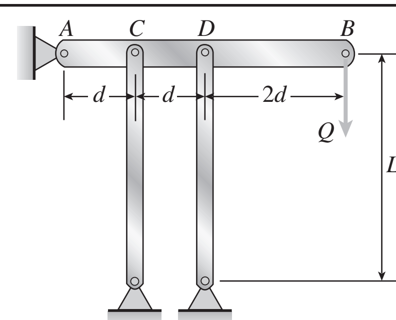
$$P_{cr} = \frac{\pi^2 EI}{L^2} = 46.264\,\text{k}$$

BEAM ACB $\quad \sum M_A = 0 \quad Q = \frac{P}{3}$

$$Q_{allow} = \frac{P_{allow}}{3} = \frac{P_{cr}}{3n} = \frac{P_{cr}}{5.4} = 8.57\,\text{k} \longleftarrow$$

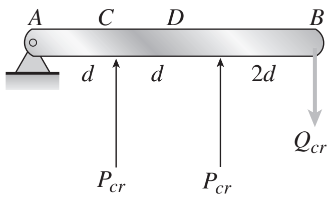
Problem 11.3-6 A horizontal beam AB is pin-supported at end A and carries a load Q at end B, as shown in the figure. The beam is supported at C and D by two identical pinned-end columns of length L. Each column has flexural rigidity EI.

What is the critical load Q_{cr}? (In other words, at what load Q_{cr} does the system collapse because of Euler buckling of the columns?)



Solution 11.3-6 Beam supported by two columns

Collapse occurs when both columns reach the critical load.

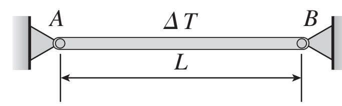


$$\sum M_A = 0 \quad Q_{cr} = \frac{3P_{cr}}{4}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \therefore Q_{cr} = \frac{3\pi^2 EI}{4L^2} \leftarrow$$

Problem 11.3-7 A slender bar AB with pinned ends and length L is held between immovable supports (see figure).

What increase ΔT in the temperature of the bar will produce buckling at the Euler load?



Solution 11.3-7 Bar with immovable pin supports

L = length $\quad A$ = cross-sectional area

I = moment of inertia $\quad E$ = modulus of elasticity

α = coefficient of thermal expansion

ΔT = uniform increase in temperature

Axial compressive force in bar (Eq. 2-17)

$$P = EA\alpha(\Delta T)$$

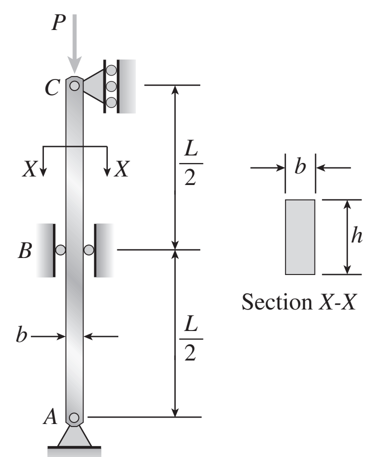
Euler load $\quad P_{cr} = \frac{\pi^2 EI}{L^2}$

Increase in temperature to produce buckling

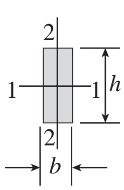
$$P = P_{cr} \quad EA\alpha(\Delta T) = \frac{\pi^2 EI}{L^2} \quad \Delta T = \frac{\pi^2 I}{\alpha A L^2} \leftarrow$$

Problem 11.3-8 A rectangular column with cross-sectional dimensions b and h is pin-supported at ends A and C (see figure). At midheight, the column is restrained in the plane of the figure but is free to deflect perpendicular to the plane of the figure.

Determine the ratio h/b such that the critical load is the same for buckling in the two principal planes of the column.



Solution 11.3-8 Column with restraint at midheight



Critical loads for buckling about axes 1-1 and 2-2:

$$P_1 = \frac{\pi^2 EI_1}{L^2} \qquad P_2 = \frac{\pi^2 EI_2}{(L/2)^2} = \frac{4\pi^2 EI_2}{L^2}$$

For equal critical loads

$$P_1 = P_2 \quad \therefore I_1 = 4I_2$$

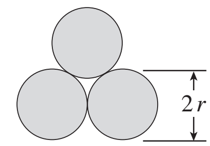
$$I_1 = \frac{bh^3}{12} \quad I_2 = \frac{hb^3}{12}$$

$$bh^3 = 4hb^3 \quad \frac{h}{b} = 2 \leftarrow$$

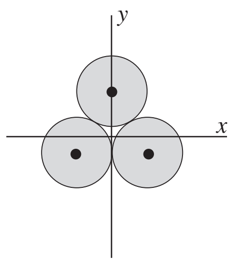
Problem 11.3-9 Three identical, solid circular rods, each of radius r and length L, are placed together to form a compression member (see the cross section shown in the figure).

Assuming pinned-end conditions, determine the critical load P_{cr} as follows: (a) The rods act independently as individual columns, and (b) the rods are bonded by epoxy throughout their lengths so that they function as a single member.

What is the effect on the critical load when the rods act as a single member?



Solution 11.3-9 **Three solid circular rods**



R = Radius L = Length

(a) Rods act independently

$$P_{cr} = \frac{\pi^2 EI}{L^2}(3) \quad I = \frac{\pi r^4}{4}$$

$$P_{cr} = \frac{3\pi^3 E r^4}{4L^2} \quad \longleftarrow$$

(b) Rods are bonded together

The x and y axes have their origin at the centroid of the cross section. Because there are three different centroidal axes of symmetry, all centroidal axes are principal axes and all centroidal moments of inertia are equal (see Section 12.9).

From Case 9, Appendix D:

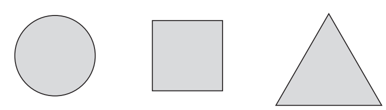
$$I = I_Y = \frac{\pi r^4}{4} + 2\left(\frac{5\pi r^4}{4}\right) = \frac{11\pi r^4}{4}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{11\pi^3 E r^4}{4L^2} \quad \longleftarrow$$

Note: Joining the rods so that they act as a single member increases the critical load by a factor of 11/3, or 3.67. ⟵

Problem 11.3-10 Three pinned-end columns of the same material have the same length and the same cross-sectional area (see figure). The columns are free to buckle in any direction. The columns have cross sections as follows: (1) a circle, (2) a square, and (3) an equilateral triangle.

Determine the ratios $P_1 : P_2 : P_3$ of the critical loads for these columns.



Solution 11.3-10 **Three pinned-end columns**

E, L, and A are the same for all three columns.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \therefore P_1 : P_2 : P_3 = I_1 : I_2 : I_3$$

(1) Circle Case 9, Appendix D

$$I = \frac{\pi d^4}{64} \quad A = \frac{\pi d^2}{4} \quad \therefore I_1 = \frac{A^2}{4\pi}$$

(2) Square Case 1, Appendix D

$$I = \frac{b^4}{12} \quad A = b^2 \quad \therefore I_2 = \frac{A^2}{12}$$

(3) Equilateral triangle Case 5, Appendix D

$$I = \frac{b^4\sqrt{3}}{96} \quad A = \frac{b^2\sqrt{3}}{4} \quad \therefore I_3 = \frac{A^2\sqrt{3}}{18}$$

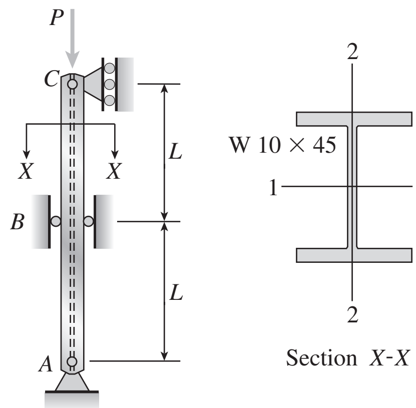
$$P_1 : P_2 : P_3 = I_1 : I_2 : I_3 = 1 : \frac{\pi}{3} : \frac{2\pi\sqrt{3}}{9}$$

$$= 1.000 : 1.047 : 1.209 \quad \longleftarrow$$

Note: For each of the above cross sections, every centroidal axis has the same moment of inertia (see Section 12.9).

Problem 11.3-11 A long slender column *ABC* is pinned at ends *A* and *C* and compressed by an axial force *P* (see figure). At the midpoint *B*, lateral support is provided to prevent deflection in the plane of the figure. The column is a steel wide-flange section (W 10 × 45) with $E = 30 \times 10^6$ psi. The distance between lateral supports is $L = 18$ ft.

Calculate the allowable load *P* using a factor of safety $n = 2.4$, taking into account the possibility of Euler buckling about either principal centroidal axis (i.e., axis 1-1 or axis 2-2).



Solution 11.3-11 **Column with restraint at midheight**

W 10 × 45 $E = 30 \times 10^6$ psi

$L = 18$ ft $= 216$ in. $I_1 = 248$ in.^{4} $I_2 = 53.4$ in.^{4}

$n = 2.4$

BUCKLING ABOUT AXIS 1-1

$$P_{cr} = \frac{\pi^2 EI_1}{(2L)^2} = 393.5\text{ k}$$

BUCKLING ABOUT AXIS 2-2

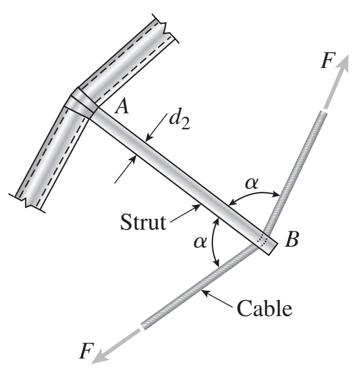
$$P_{cr} = \frac{\pi^2 EI_2}{L^2} = 338.9\text{ k}$$

ALLOWABLE LOAD

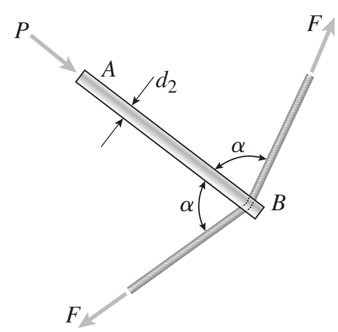
$$P_{allow} = \frac{P_{cr}}{n} = \frac{338.9\text{ k}}{2.4} = 141\text{ k} \quad \longleftarrow$$

Problem 11.3-12 The multifaceted glass roof over the lobby of a museum building is supported by the use of pretensioned cables. At a typical joint in the roof structure, a strut *AB* is compressed by the action of tensile forces *F* in a cable that makes an angle $\alpha = 75°$ with the strut (see figure). The strut is a circular tube of aluminum ($E = 72$ GPa) with outer diameter $d_2 = 50$ mm and inner diameter $d_1 = 40$ mm. The strut is 1.0 m long and is assumed to be pin-connected at both ends.

Using a factor of safety $n = 2.5$ with respect to the critical load, determine the allowable force *F* in the cable.



Solution 11.3-12 **Strut and cable**



P = compressive force in strut

F = tensile force in cable

α = angle between strut and cable
 = 75°

PROPERTIES OF STRUT $E = 72$ GPa

$d_2 = 50$ mm $d_1 = 40$ mm $L = 1.0$ m

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 181.13 \times 10^3\text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 128.71\text{ kN}$$

$$P_{allow} = \frac{P_{cr}}{n} = \frac{128.71\text{ kN}}{2.5} = 51.49\text{ kN}$$

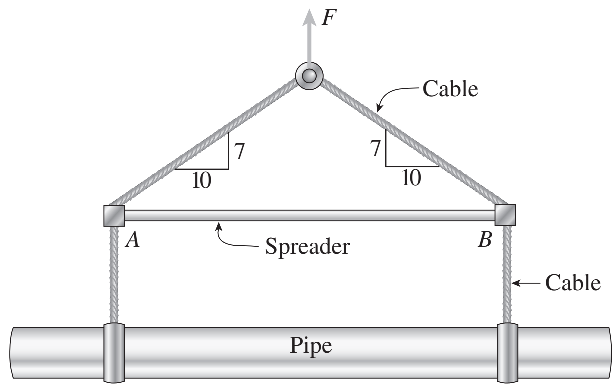
EQUILIBRIUM OF JOINT *B*

$P = 2F\cos 75°$

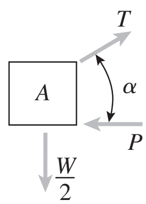
$$\therefore\ F_{allow} = \frac{P_{allow}}{2\cos 75°} = 99.5\text{ kN} \quad \longleftarrow$$

Problem 11.3-13 The hoisting arrangement for lifting a large pipe is shown in the figure. The spreader is a steel tubular section with outer diameter 2.75 in. and inner diameter 2.25 in. Its length is 8.5 ft and its modulus of elasticity is 29×10^6 psi.

Based upon a factor of safety of 2.25 with respect to Euler buckling of the spreader, what is the maximum weight of pipe that can be lifted? (Assume pinned conditions at the ends of the spreader.)



Solution 11.3-13 Hoisting arrangement for a pipe



T = tensile force in cable
P = compressive force in spreader
W = weight of pipe

$$\tan\alpha = \frac{7}{10}$$

PROPERTIES OF SPREADER $E = 29 \times 10^6$ psi

$d_2 = 2.75$ in. $d_1 = 2.25$ in. $L = 8.5$ ft $= 102$ in.

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.549 \text{ in.}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 42.61 \text{ k}$$

$$P_{allow} = \frac{P_{cr}}{n} = \frac{42.61 \text{ k}}{2.25} = 18.94 \text{ k}$$

EQUILIBRIUM OF JOINT A

$$\sum F_{horiz} = 0 \qquad -P + T\cos\alpha = 0$$

$$\sum F_{vert} = 0 \qquad T\sin\alpha - \frac{W}{2} = 0$$

SOLVE THE EQUATION

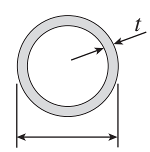
$$W = 2P\tan\alpha$$

MAXIMUM WEIGHT OF PIPE

$$W_{max} = 2P_{allow}\tan\alpha = 2(18.94 \text{ k})(0.7)$$
$$= 26.5 \text{ k} \longleftarrow$$

Problem 11.3-14 A pinned-end strut of aluminum ($E = 72$ GPa) with length $L = 1.8$ m is constructed of circular tubing with outside diameter $d = 50$ mm (see figure). The strut must resist an axial load $P = 18$ kN with a factor of safety $n = 2.0$ with respect to the critical load.

Determine the required thickness t of the tube.



Solution 11.3-14 **Aluminum strut**

$E = 72$ GPa $\quad L = 1.8$ m

Outer diameter $d = 50$ mm

$t =$ thickness

Inner diameter $= d - 2t$

$P = 18$ kN $\quad n = 2.0$

CRITICAL LOAD $\quad P_{cr} = nP = (2.0)(18 \text{ kN}) = 36$ kN

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \therefore I = \frac{P_{cr}L^2}{\pi^2 E} = 164.14 \times 10^3 \text{ mm}^4$$

MOMENT OF INERTIA

$$I = \frac{\pi}{64}[d^4\ 2\ (d\ 2\ 2t)^4] = 164.14\ 3\ 10^3 \text{ mm}^4$$

REQUIRED THICKNESS

$$d^4 - (d - 2t)^4 = 3.3438 \times 10^6 \text{ mm}^4$$

$$(d - 2t)^4 = (50 \text{ mm})^4 - 3.3438 \times 10^6 \text{ mm}^4$$

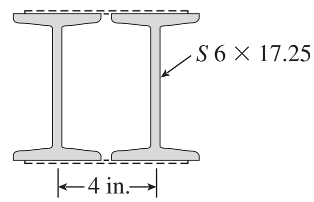
$$= 2.9062 \times 10^6 \text{ mm}^4$$

$$d - 2t = 41.289 \text{ mm}$$

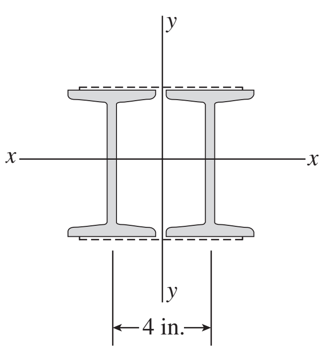
$$2t = 50 \text{ mm} - 41.289 \text{ mm} = 8.711 \text{ mm}$$

$$t_{min} = 4.36 \text{ mm} \quad \longleftarrow$$

Problem 11.3-15 The cross section of a column built up of two steel I-beams (S 6 × 17.25 sections) is shown in the figure on the next page. The beams are connected by spacer bars, or *lacing*, to ensure that they act together as a single column. (The lacing is represented by dashed lines in the figure.) The column is assumed to have pinned ends and may buckle in any direction. Assuming $E = 30 \times 10^6$ psi and $L = 27.5$ ft, calculate the critical load P_{cr} for the column.



Solution 11.3-15 **Column of two steel beams**



S 6 × 17.25

$E = 30 \times 10^6$ psi

$L = 27.5$ ft $= 330$ in.

$I_1 = 26.3$ in.^{4}

$I_2 = 2.31$ in.^{4}

$A = 5.07$ in.^{2}

COMPOSITE COLUMN $\quad I_x = 2I_1 = 52.6$ in.^{4}

$$I_y = 2(I_2 + Ad^2) \qquad d = \frac{4 \text{ in.}}{2} = 2 \text{ in.}$$

$$I_y = 2[2.31 \text{ in.}^4 + (5.07 \text{ in.}^2)(2 \text{ in.})^2]$$

$$= 45.18 \text{ in.}^4 \qquad I_y < I_x$$

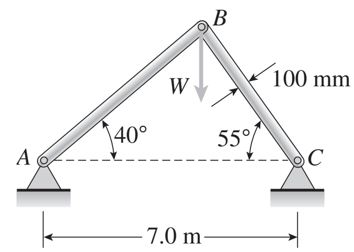
$\therefore$ Buckling occurs about the y axis.

CRITICAL LOAD

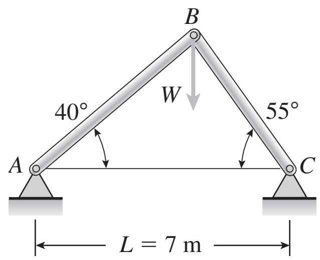
$$P_{cr} = \frac{\pi^2 EI_y}{L^2} = 123 \text{ k} \quad \longleftarrow$$

Problem 11.3-16 The truss *ABC* shown in the figure supports a vertical load W at joint B. Each member is a slender circular steel pipe (E = 200 GPa) with outside diameter 100 mm and wall thickness 6.0 mm. The distance between supports is 7.0 m. Joint B is restrained against displacement perpendicular to the plane of the truss.

Determine the critical value W_{cr} of the load.



Solution 11.3-16 **Truss *ABC* with load *W***



STEEL PIPES *AB* AND *BC*

E = 200 GPa L = 7.0 m

d_2 = 100 mm t = 6.0 mm

$d_1 = d_2 - 2t$ = 88 mm

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.965 \times 10^6 \text{ mm}^4$$

LENGTHS OF MEMBERS *AB* AND *BC*

use the law of sines (see Appendix C)

$$L_{AB} = L\left(\frac{\sin 55°}{\sin 85°}\right) = 5.756 \text{ m}$$

$$L_{BC} = L\left(\frac{\sin 40°}{\sin 85°}\right) = 4.517 \text{ m}$$

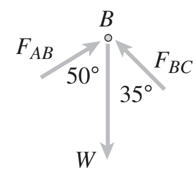
Buckling occurs when either member reaches its critical load.

CRITICAL LOADS

$$(P_{cr})_{AB} = \frac{\pi^2 EI}{L_{AB}^2} = 117.1 \text{ kN}$$

$$(P_{cr})_{BC} = \frac{\pi^2 EI}{L_{BC}^2} = 190.1 \text{ kN}$$

FREE-BODY DIAGRAM OF JOINT *B*



$\sum F_{horiz} = 0$ $F_{AB} \sin 50° - F_{BC} \sin 35° = 0$

$\sum F_{vert} = 0$ $F_{AB} \cos 50° - F_{BC} \cos 35° - W = 0$

SOLVE THE TWO EQUATIONS

$W = 1.7368\, F_{AB}$ $W = 1.3004\, F_{BC}$

CRITICAL VALUE OF THE LOAD *W*

Based on member *AB*: $W_{cr} = 1.7368\,(P_{cr})_{AB}$
$= 203$ kN

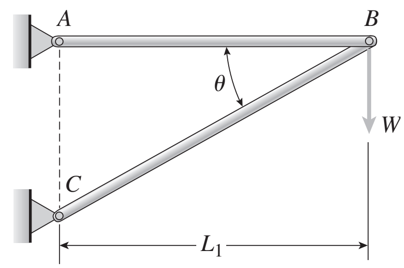
Based on member *BC*: $W_{cr} = 1.3004\,(P_{cr})_{BC}$
$= 247$ kN

lower load governs. Member *AB* buckler.

W_{cr} = 203 kN ←

Problem 11.3-17 A truss ABC supports a load W at joint B, as shown in the figure. The length L_1 of member AB is fixed, but the length of strut BC varies as the angle θ is changed. Strut BC has a solid circular cross section. Joint B is restrained against displacement perpendicular to the plane of the truss.

Assuming that collapse occurs by Euler buckling of the strut, determine the angle θ for minimum weight of the strut.



Solution 11.3-17 **Truss ABC (minimum weight)**

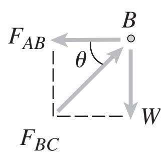
LENGTHS OF MEMBERS

$L_{AB} = L_1$ (a constant)

$$L_{BC} = \frac{L_1}{\cos\theta} \text{(angle } \theta \text{ is variable)}$$

Strut BC may buckle.

FREE-BODY DIAGRAM OF JOINT B



$$\sum F_{\text{vert}} = 0 \qquad F_{BC}\sin\theta - W = 0$$

$$F_{BC} = \frac{W}{\sin\theta}$$

STRUT BC (SOLID CIRCULAR BAR)

$$A = \frac{\pi d^2}{4} \qquad I = \frac{\pi d^4}{64} \qquad \therefore\ I = \frac{A^2}{4\pi}$$

$$P_{cr} = \frac{\pi^2 EI}{L_{BC}^2} = \frac{\pi E A^2 \cos^2\theta}{4L_1^2}$$

$$F_{BC} = P_{cr} \quad \text{or} \quad \frac{W}{\sin\theta} = \frac{\pi E A^2 \cos^2\theta}{4L_1^2}$$

Solve for area A: $A = \dfrac{2L_1}{\cos\theta}\left(\dfrac{W}{\pi E\sin\theta}\right)^{1/2}$

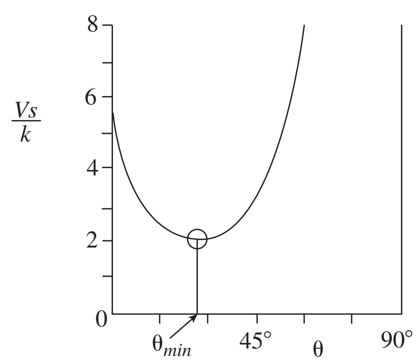
For minimum weight, the volume V_S of the strut must be a minimum.

$$V_S = AL_{BC} = \frac{AL_1}{\cos\theta} = \frac{2L_1^2}{\cos^2\theta}\left(\frac{W}{\pi E\sin\theta}\right)^{1/2}$$

All terms are constants except $\cos\theta$ and $\sin\theta$. Therefore, we can write V_S in the following form:

$$V_S = \frac{k}{\cos^2\theta\sqrt{\sin\theta}} \quad \text{where } k \text{ is a constant.}$$

GRAPH OF $\dfrac{V_S}{k}$



$\theta_{\min}$ = angle for minimum volume (and minimum weight)

For minimum weight, the term $\cos^2\theta\sqrt{\sin\theta}$ must be a maximum.

For a maximum value, the derivative with respect to θ equals zero.

Therefore, $\dfrac{d}{d\theta}(\cos^2\theta\sqrt{\sin\theta}) = 0$

Taking the derivative and simplifying, we get
$\cos^2\theta - 4\sin^2\theta = 0$

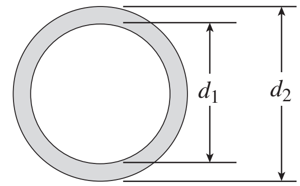
or $1 - 4\tan^2\theta = 0$ and $\tan\theta = \dfrac{1}{2}$

Columns with Other Support Conditions

The problems for Section 11.4 are to be solved using the assumptions of ideal, slender, prismatic, linearly elastic columns (Euler buckling). Buckling occurs in the plane of the figure unless stated otherwise.

Problem 11.4-1 An aluminum pipe column ($E = 10{,}400$ ksi) with length $L = 10.0$ ft has inside and outside diameters $d_1 = 5.0$ in. and $d_2 = 6.0$ in., respectively (see figure). The column is supported only at the ends and may buckle in any direction.

Calculate the critical load P_{cr} for the following end conditions: (1) pinned-pinned, (2) fixed-free, (3) fixed-pinned, and (4) fixed-fixed.



Probs. 11.4-1 and 11.4-2

Solution 11.4-1 Aluminum pipe column

$d_2 = 6.0$ in. $\quad d_1 = 5.0$ in. $\quad E = 10{,}400$ ksi

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 32.94 \text{ in.}^4$$

$L = 10.0 \text{ ft} = 120$ in.

(1) Pinned-pinned

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(10{,}400 \text{ ksi})(32.94 \text{ in.}^4)}{(120 \text{ in.})^2} = 235 \text{ k} \longleftarrow$$

(2) Fixed-free $\quad P_{cr} = \dfrac{\pi^2 EI}{4L^2} = 58.7 \text{ k} \longleftarrow$

(3) Fixed-pinned $\quad P_{cr} = \dfrac{2.046\,\pi^2 EI}{L^2} = 480 \text{ k} \longleftarrow$

(4) Fixed-fixed $\quad P_{cr} = \dfrac{4\pi^2 EI}{L^2} = 939 \text{ k} \longleftarrow$

Problem 11.4-2 Solve the preceding problem for a steel pipe column ($E = 210$ GPa) with length $L = 1.2$ m, inner diameter $d_1 = 36$ mm, and outer diameter $d_2 = 40$ mm.

Solution 11.4-2 Steel pipe column

$d_2 = 40$ mm $\quad d_1 = 36$ mm $\quad E = 210$ GPa

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 43.22 \times 10^3 \text{ mm}^4 \quad L = 1.2 \text{ m}$$

(1) Pinned-pinned $\quad P_{cr} = \dfrac{\pi^2 EI}{L^2} = 62.2 \text{ kN} \longleftarrow$

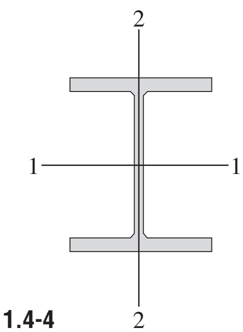
(2) Fixed-free $\quad P_{cr} = \dfrac{\pi^2 EI}{4L^2} = 15.6 \text{ kN} \longleftarrow$

(3) Fixed-pinned $\quad P_{cr} = \dfrac{2.046\,\pi^2 EI}{L^2} = 127 \text{ kN} \longleftarrow$

(4) Fixed-fixed $\quad P_{cr} = \dfrac{4\pi^2 EI}{L^2} = 249 \text{ kN} \longleftarrow$

Problem 11.4-3 A wide-flange steel column ($E = 30 \times 10^6$ psi) of W 12 × 87 shape (see figure) has length $L = 28$ ft. It is supported only at the ends and may buckle in any direction.

Calculate the allowable load P_{allow} based upon the critical load with a factor of safety $n = 2.5$. Consider the following end conditions: (1) pinned-pinned, (2) fixed-free, (3) fixed-pinned, and (4) fixed-fixed.



Probs. 11.4-3 and 11.4-4

Solution 11.4-3 **Wide-flange column**

W 12 × 87 $E = 30 \times 10^6$ psi

$L = 28$ ft $= 336$ in. $n = 2.5$ $I_2 = 241$ in.^{4}

(1) Pinned-pinned

$$P_{allow} = \frac{P_{cr}}{n} = \frac{\pi^2 EI_2}{nL^2} = 253\text{ k} \longleftarrow$$

(2) Fixed-free

$$P_{allow} = \frac{\pi^2 EI_2}{4nL^2} = 63.2\text{ k} \longleftarrow$$

(3) Fixed-pinned

$$P_{allow} = \frac{2.046\pi^2 EI_2}{nL^2} = 517\text{ k} \longleftarrow$$

(4) Fixed-fixed

$$P_{allow} = \frac{4\pi^2 EI_2}{nL^2} = 1011\text{ k} \longleftarrow$$

Problem 11.4-4 Solve the preceding problem for a W 10 × 60 shape with length $L = 24$ ft.

Solution 11.4-4 **Wide-flange column**

W 10 × 60 $E = 30 \times 10^6$ psi

$L = 24$ ft $= 288$ in. $n = 2.5$ $I_2 = 116$ in.^{4}

(1) Pinned-pinned

$$P_{allow} = \frac{P_{cr}}{n} = \frac{\pi^2 EI_2}{nL^2} = 166\text{ k} \longleftarrow$$

(2) Fixed-free

$$P_{allow} = \frac{\pi^2 EI_2}{4nL^2} = 41.4\text{ k} \longleftarrow$$

(3) Fixed-pinned

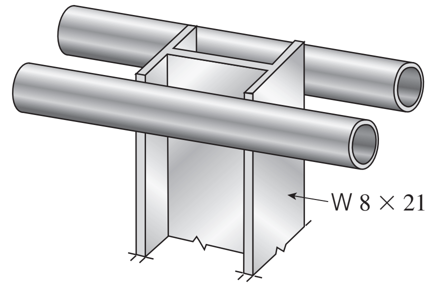
$$P_{allow} = \frac{2.046\pi^2 EI_2}{nL^2} = 339\text{ k} \longleftarrow$$

(4) Fixed-fixed

$$P_{allow} = \frac{4\pi^2 EI_2}{nL^2} = 663\text{ k} \longleftarrow$$

Problem 11.4-5 The upper end of a W 8 × 21 wide-flange steel column ($E = 30 \times 10^3$ ksi) is supported laterally between two pipes (see figure). The pipes are not attached to the column, and friction between the pipes and the column is unreliable. The base of the column provides a fixed support, and the column is 13 ft long.

Determine the critical load for the column, considering Euler buckling in the plane of the web and also perpendicular to the plane of the web.

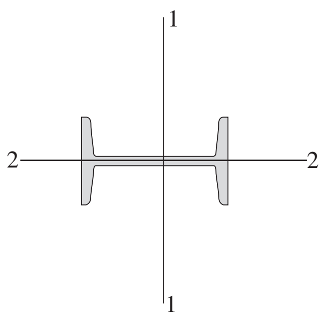


Solution 11.4-5 **Wide-flange steel column**

W 8 × 21 $E = 30 \times 10^3$ ksi

$L = 13$ ft $= 156$ in. $I_1 = 75.3$ in.^{4}

$I_2 = 9.77$ in.^{4}



AXIS 1-1 (FIXED-FREE)

$$P_{cr} = \frac{\pi^2 EI_1}{4L^2} = 229\,\text{k}$$

AXIS 2-2 (FIXED-PINNED)

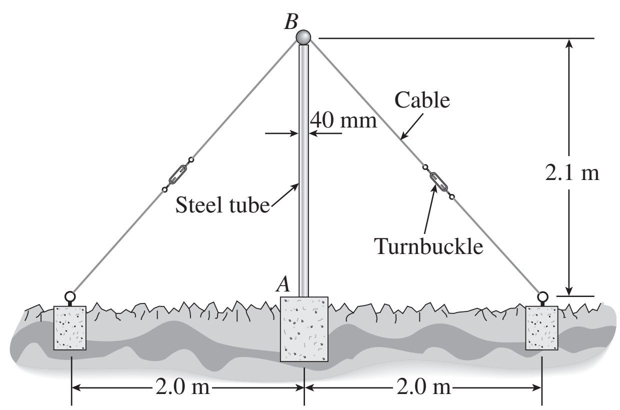
$$P_{cr} = \frac{2.046\pi^2 EI_2}{L^2} = 243\,\text{k}$$

Buckling about axis 1-1 governs.

$P_{cr} = 229$ k ⟵

Problem 11.4-6 A vertical post *AB* is embedded in a concrete foundation and held at the top by two cables (see figure). The post is a hollow steel tube with modulus of elasticity 200 GPa, outer diameter 40 mm, and thickness 5 mm. The cables are tightened equally by turnbuckles.

If a factor of safety of 3.0 against Euler buckling in the plane of the figure is desired, what is the maximum allowable tensile force T_{allow} in the cables?



Solution 11.4-6 **Steel tube**

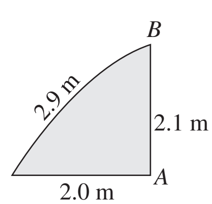
$E = 200$ GPa $d_2 = 40$ mm $d_1 = 30$ mm
$L = 2.1$ m $n = 3.0$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 85{,}903\,\text{mm}^4$$

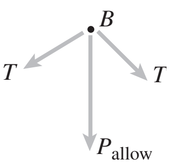
Buckling in the plane of the figure means fixed-pinned end conditions.

$$P_{cr} = \frac{2.046\pi^2 EI}{L^2} = 78.67\text{ kN}$$

$$P_{allow} = \frac{P_{cr}}{n} = \frac{78.67\,\text{kN}}{3.0} = 26.22\,\text{kN}$$



FREE-BODY DIAGRAM OF JOINT B



T = tensile force in each cable
P_{allow} = compressive force in tube

EQUILIBRIUM

$$\sum F_{vert} = 0 \quad P_{allow} - 2T\left(\frac{2.1\text{ m}}{2.9\text{ m}}\right) = 0$$

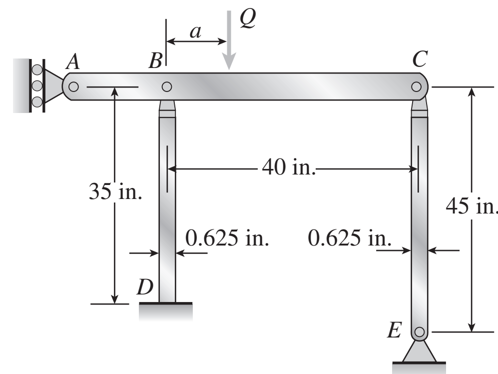
ALLOWABLE FORCE IN CABLES

$$T_{allow} = (P_{allow})\left(\frac{1}{2}\right)\left(\frac{2.9\text{ m}}{2.1\text{ m}}\right) = 18.1\text{ kN} \quad \longleftarrow$$

Problem 11.4-7 The horizontal beam ABC shown in the figure is supported by columns BD and CE. The beam is prevented from moving horizontally by the roller support at end A, but vertical displacement at end A is free to occur. Each column is pinned at its upper end to the beam, but at the lower ends, support D is fixed and support E is pinned. Both columns are solid steel bars ($E = 30 \times 10^6$ psi) of square cross section with width equal to 0.625 in. A load Q acts at distance a from column BD.

(a) If the distance $a = 12$ in., what is the critical value Q_{cr} of the load?

(b) If the distance a can be varied between 0 and 40 in., what is the maximum possible value of Q_{cr}? What is the corresponding value of the distance a?



Solution 11.4-7 Beam supported by two columns

COLUMN BD $\quad E = 30 \times 10^6$ psi $\quad L = 35$ in.

$$b = 0.625\text{ in.} \quad I = \frac{b^4}{12} = 0.012716\text{ in.}^4$$

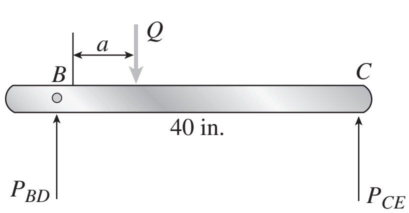
$$P_{cr} = \frac{2.046\,\pi^2 EI}{L^2} = 6288\text{ lb}$$

COLUMN CE $\quad E = 30 \times 10^6$ psi $\quad L = 45$ in.

$$b = 0.625\text{ in.} \quad I = \frac{b^4}{12} = 0.012716\text{ in.}^4$$

$$P_{CR} = \frac{\pi^2 EI}{L^2} = 1859\text{ lb}$$

(a) FIND Q_{cr} IF $a = 12$ in.



$$P_{BD} = \frac{28}{40}Q = \frac{7}{10}Q \quad Q = \frac{10}{7}P_{BD}$$

$$P_{CE} = \frac{12}{40}Q = \frac{3}{10}Q \quad Q = \frac{10}{3}P_{CE}$$

If column BD buckles: $Q = \frac{10}{7}(6288\text{ lb}) = 8980\text{ lb}$

If column CE buckles: $Q = \frac{10}{3}(1859\text{ lb}) = 6200\text{ lb}$

$$\therefore\ Q_{cr} = 6200\text{ lb} \quad \longleftarrow$$

(b) MAXIMUM VALUE OF Q_{CR}

Both columns buckle simultaneously.
$P_{BD} = 6288$ lb $\quad P_{CE} = 1859$ lb

$$\sum F_{vert} = 0 \quad Q_{CR} = P_{BD} + P_{CE} = 8150\text{ lb} \quad \longleftarrow$$

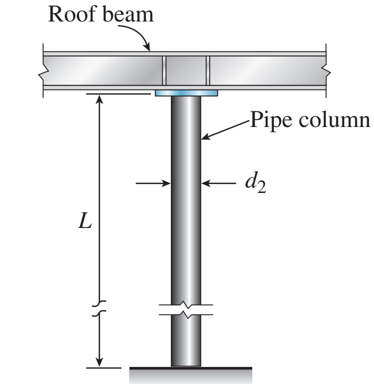
$$\sum M_B = 0 \quad Q_{CR}(a) = P_{CE}(40\text{ in.})$$

$$a = \frac{P_{CE}(40\text{ in.})}{Q_{cr}} = \frac{(1859\text{ lb})(40\text{ in.})}{P_{BD} + P_{CE}}$$

$$= \frac{(1859\text{ lb})(40\text{ in.})}{6288\text{ lb} + 1859\text{ lb}} = 9.13\text{ in.} \quad \longleftarrow$$

Problem 11.4-8 The roof beams of a warehouse are supported by pipe columns (see figure on the next page) having outer diameter $d_2 = 100$ mm and inner diameter $d_1 = 90$ mm. The columns have length $L = 4.0$ m, modulus $E = 210$ GPa, and fixed supports at the base.

Calculate the critical load P_{cr} of one of the columns using the following assumptions: (1) the upper end is pinned and the beam prevents horizontal displacement; (2) the upper end is fixed against rotation and the beam prevents horizontal displacement; (3) the upper end is pinned but the beam is free to move horizontally; and (4) the upper end is fixed against rotation but the beam is free to move horizontally.



Solution 11.4-8 **Pipe column (with fixed base)**

$E = 210$ GPa $\quad L = 4.0$ m

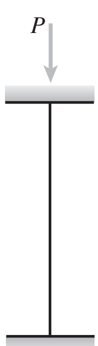
$d_2 = 100$ mm $\quad I = \dfrac{\pi}{64}(d_2^4 - d_1^4) = 1688 \times 10^3\ \text{mm}^4$

$d_1 = 90$ mm

(1) Upper end is pinned (with no horizontal displacement)

$$P_{cr} = \frac{2.046\pi^2 EI}{L^2} = 447\ \text{kN} \quad \longleftarrow$$

(2) Upper end is fixed (with no horizontal displacement)

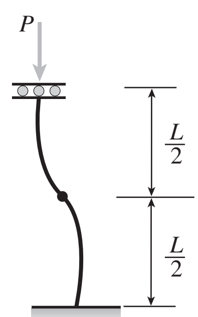


$$P_{cr} = \frac{4\pi^2 EI}{L^2} = 875\ \text{kN} \quad \longleftarrow$$

(3) Upper end is pinned (but no horizontal restraint)

$$P_{cr} = \frac{\pi^2 EI}{4L^2} = 54.7\ \text{kN} \quad \longleftarrow$$

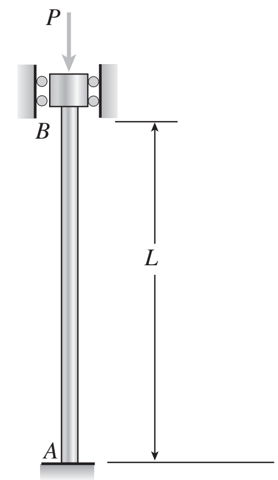
(4) Upper end is guided
(no rotation; no horizontal restraint)



The lower half of the column is in the same condition as Case (3) above.

$$P_{cr} = \frac{\pi^2 EI}{4(L/2)^2} = \frac{\pi^2 EI}{L^2} = 219\ \text{kN} \quad \longleftarrow$$

Problem 11.4-9 Determine the critical load P_{cr} and the equation of the buckled shape for an ideal column with ends fixed against rotation (see figure) by solving the differential equation of the deflection curve. (See also Fig. 11-17.)



Solution 11.4-9 Fixed-end column

v = deflection in the y direction

DIFFERENTIAL EQUATION (EQ.11-3)

$$EIv'' = M = M_0 - Pv \qquad k^2 = \frac{P}{EI}$$

$$v'' + k^2 v = \frac{M_0}{EI}$$

GENERAL SOLUTION

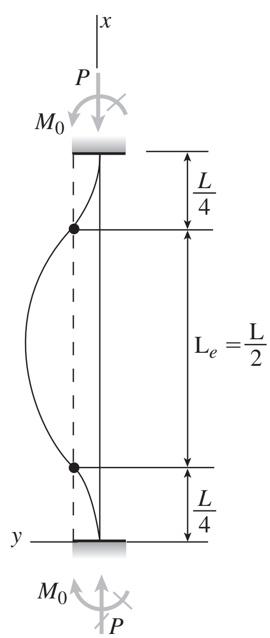
$$v = C_1 \sin kx + C_2 \cos kx + \frac{M_0}{P}$$

B.C. 1 $v(0) = 0 \qquad \therefore C_2 = -\frac{M_0}{P}$

$$v' = C_1 k \cos kx - C_2 k \sin kx$$

B.C. 2 $v'(0) = 0 \qquad \therefore C_1 = 0$

$$v = \frac{M_0}{P}(1 - \cos kx)$$



BUCKLING EQUATION

B.C. 3 $v(L) = 0 \qquad 0 = \frac{M_0}{P}(1 - \cos kL)$

$\therefore \cos kL = 1 \quad$ and $\quad kL = 2\pi$

CRITICAL LOAD

$$k^2 = \left(\frac{2\pi}{L}\right)^2 = \frac{4\pi^2}{L^2} \qquad \frac{P}{EI} = \frac{4\pi^2}{L^2}$$

$$P_{cr} = \frac{4\pi^2 EI}{L^2} \quad \longleftarrow$$

BUCKLED MODE SHAPE

Let δ = deflection at midpoint $\left(x = \frac{L}{2}\right)$

$$v\left(\frac{L}{2}\right) = \delta = \frac{M_0}{P}\left(1 - \cos\frac{kL}{2}\right)$$

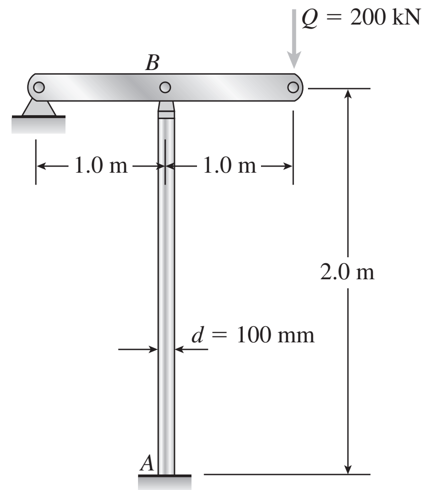
$$\frac{kL}{2} = \pi \qquad \therefore \qquad \delta = \frac{M_0}{P}(1 - \cos \pi)$$

$$= \frac{2M_0}{P} \qquad \frac{M_0}{P} = \frac{\delta}{2}$$

$$v = \frac{\delta}{2}\left(1 - \cos\frac{2\pi x}{L}\right) \quad \longleftarrow$$

Problem 11.4-10 An aluminum tube AB of circular cross section is fixed at the base and pinned at the top to a horizontal beam supporting a load $Q = 200$ kN (see figure).

Determine the required thickness t of the tube if its outside diameter d is 100 mm and the desired factor of safety with respect to Euler buckling is $n = 3.0$. (Assume $E = 72$ GPa.)



Solution 11.4-10 Aluminum tube

End conditions: Fixed-pinned

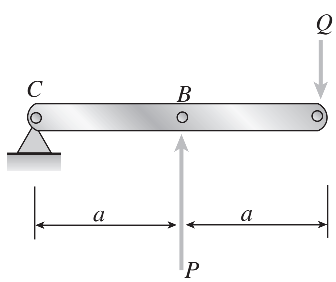
$E = 72$ GPa $\quad L = 2.0$ m $\quad n = 3.0$
$d_2 = 100$ mm $\quad t =$ thickness (mm)
$d_1 = 100$ mm $- 2t$

MOMENT OF INERTIA (mm^4)

$$I = \frac{\pi}{64}(d_2^4 - d_1^4)$$

$$= \frac{\pi}{64}[(100)^4 - (100 - 2t)^4] \qquad (1)$$

HORIZONTAL BEAM



$Q = 200$ kN
$P =$ compressive force in tube
$\sum M_c = 0 \quad \mathrm{Pa} - 2Qa = 0$

$$Q = \frac{P}{2} \quad \therefore\ P = 2Q = 400\ \text{kN}$$

ALLOWABLE FORCE P

$$P_{\text{allow}} = \frac{P_{\text{cr}}}{n} = \frac{2.046\pi^2 EI}{nL^2} \qquad (2)$$

MOMENT OF INERTIA

$$I = \frac{nL^2 P_{\text{allow}}}{2.046\pi^2 E} = \frac{(3.0)(2.0\,\text{m})^2(400\,\text{kN})}{(2.046)(\pi^2)(72\,\text{GPa})}$$

$$= 3.301 \times 10^{-6}\ \text{m}^4 = 3.301 \times 10^6\ \text{mm}^4 \qquad (3)$$

EQUATE (1) AND (3):

$$\frac{\pi}{64}[(100)^4 - (100 - 2t)^4] = 3.301 \times 10^6$$

$$(100 - 2t)^4 = 32.74 \times 10^6\ \text{mm}^4$$

$$100 - 2t = 75.64\ \text{mm} \qquad t_{\min} = 12.2\ \text{mm} \longleftarrow$$

Problem 11.4-11 The frame ABC consists of two members AB and BC that are rigidly connected at joint B, as shown in part (a) of the figure. The frame has pin supports at A and C. A concentrated load P acts at joint B, thereby placing member AB in direct compression.

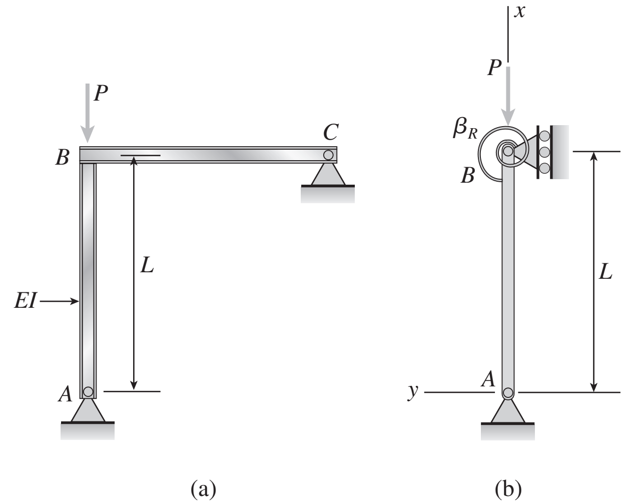
To assist in determining the buckling load for member AB, we represent it as a pinned-end column, as shown in part (b) of the figure. At the top of the column, a rotational spring of stiffness β_R represents the restraining action of the horizontal beam BC on the column (note that the horizontal beam provides resistance to rotation of joint B when the column buckles). Also, consider only bending effects in the analysis (i.e., disregard the effects of axial deformations).

(a) By solving the differential equation of the deflection curve, derive the following buckling equation for this column:

$$\frac{\beta_R L}{EI}(kL \cot kL - 1) - k^2L^2 = 0$$

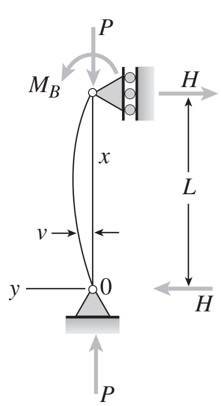
in which L is the length of the column and EI is its flexural rigidity.

(b) For the particular case when member BC is identical to member AB, the rotational stiffness β_R equals $3EI/L$ (see Case 7, Table G-2, Appendix G). For this special case, determine the critical load P_{cr}.



Solution 11.4-11 **Column AB with elastic support at B**

FREE-BODY DIAGRAM OF COLUMN



v = deflection in the y direction

M_B = moment at end B

θ_B = angle of rotation at end B (positive clockwise)

$M_B = \beta_R\theta_B$

H = horizontal reactions at ends A and B

EQUILIBRIUM

$$\sum M_0 = \sum M_A = 0 \quad M_B - HL = 0$$

$$H = \frac{M_B}{L} = \frac{\beta_R\theta_B}{L}$$

DIFFERENTIAL EQUATION (EQ. 11-3)

$$EIv'' = M = Hx - Pv \quad k^2 = \frac{P}{EI}$$

$$v'' + k^2v = \frac{\beta_R\theta_B}{LEI}x$$

GENERAL SOLUTION

$$v = C_1 \sin kx + C_2 \cos kx + \frac{\beta_R\theta_B}{PL}x$$

B.C. 1 $\quad v(0) = 0 \quad \therefore C_2 = 0$

B.C. 2 $\quad v(L) = 0 \quad \therefore C_1 = -\frac{\beta_R\theta_B}{P\sin kL}$

$$v = C_1 \sin kx + \frac{\beta_R\theta_B}{PL}x$$

$$v' = C_1 k \cos kx + \frac{\beta_R\theta_B}{PL}$$

(a) BUCKLING EQUATION

B.C. 3 $v'(L) = -\theta_B$

$$\therefore\ -\theta_B = -\frac{\beta_R\theta_B}{P\sin kL}(k\cos kL) + \frac{\beta_R\theta_B}{PL}$$

Cancel θ_B and multiply by PL:

$$-PL = -\beta_R\, kL\cot kL + \beta_R$$

Substitute $P = k^2EI$ and rearrange:

$$\frac{\beta_R L}{EI}(kL\cot kL - 1) - k^2L^2 = 0 \quad \longleftarrow$$

(b) CRITICAL LOAD FOR $\beta_R = 3EI/L$

$3(kL\cot kL - 1) - (kL)^2 = 0$

Solve numerically for kL: $kL = 3.7264$

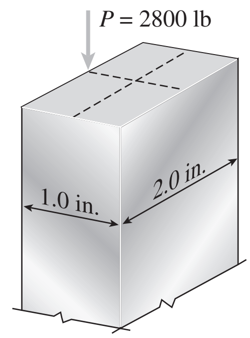
$$P_{cr} = k^2EI = (kL)^2\left(\frac{EI}{L^2}\right) = 13.89\frac{EI}{L^2} \quad \longleftarrow$$

Columns with Eccentric Axial Loads

When solving the problems for Section 11.5, assume that bending occurs in the principal plane containing the eccentric axial load.

Problem 11.5-1 An aluminum bar having a rectangular cross section (2.0 in. × 1.0 in.) and length $L = 30$ in. is compressed by axial loads that have a resultant $P = 2800$ lb acting at the midpoint of the long side of the cross section (see figure).

Assuming that the modulus of elasticity E is equal to 10×10^6 psi and that the ends of the bar are pinned, calculate the maximum deflection δ and the maximum bending moment M_{max}.



Solution 11.5-1 **Bar with rectangular cross section**

$b = 2.0$ in. $h = 1.0$ in. $L = 30$ in.
$P = 2800$ lb $e = 0.5$ in. $E = 10 \times 10^6$ psi

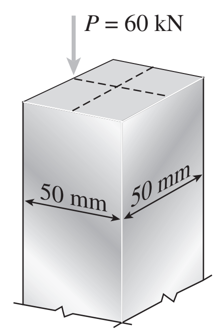
$$I = \frac{bh^3}{12} = 0.1667\text{ in.}^4 \quad kL = L\sqrt{\frac{P}{EI}} = 1.230 \quad \longleftarrow$$

Eq. (11-51): $\delta = e\left(\sec\frac{kL}{2} - 1\right) = 0.112\text{ in.}$ ⟵

Eq. (11-56): $M_{max} = Pe\sec\frac{kL}{2}$
$= 1710$ lb-in. ⟵

Problem 11.5-2 A steel bar having a square cross section (50 mm × 50 mm) and length $L = 2.0$ m is compressed by axial loads that have a resultant $P = 60$ kN acting at the midpoint of one side of the cross section (see figure).

Assuming that the modulus of elasticity E is equal to 210 GPa and that the ends of the bar are pinned, calculate the maximum deflection δ and the maximum bending moment M_{max}.



Solution 11.5-2 **Bar with square cross section**

$b = 50$ mm. $L = 2$ m. $P = 60$ kN $e = 25$ mm

$E = 210$ GPa $\quad I = \frac{b^4}{12} = 520.8 \times 10^3\ \text{mm}^4$

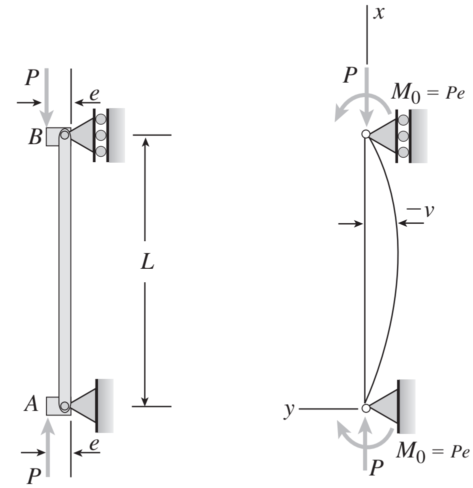
$$kL = L\sqrt{\frac{P}{EI}} = 1.481$$

Eq. (11-51): $\delta = e\left(\sec\frac{kL}{2} - 1\right) = 8.87\text{ mm}$ ⟵

Eq. (11-56): $M_{max} = Pe\sec\frac{kL}{2} = 2.03\text{ kN}\cdot\text{m}$ ⟵

Problem 11.5-3 Determine the bending moment M in the pinned-end column with eccentric axial loads shown in the figure. Then plot the bending-moment diagram for an axial load $P = 0.3P_{cr}$.

Note: Express the moment as a function of the distance x from the end of the column, and plot the diagram in nondimensional form with M/Pe as ordinate and x/L as abscissa.



Probs. 11.5-3, 11.5-4, and 11.5-5

Solution 11.5-3 Column with eccentric loads

Column has pinned ends.

Use Eq. (11-49):

$$v = -e\left(\tan\frac{kL}{2}\sin kx + \cos kx - 1\right)$$

From Eq. (11-45): $M = Pe - Pv$

$$\therefore M = Pe\left(\tan\frac{kL}{2}\sin kx + \cos kx\right) \longleftarrow$$

For $P = 0.3\ P_{cr}$:

From Eq. (11-52): $kL = \pi\sqrt{\dfrac{P}{P_{cr}}} = \pi\sqrt{0.3}$

$= 1.7207$

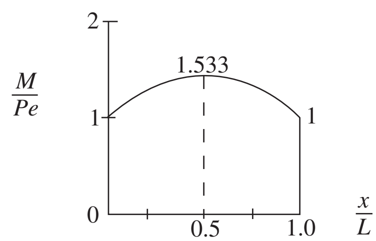
$$\frac{M}{Pe} = \left(\tan\frac{1.7207}{2}\right)\left(\sin 1.7207\frac{x}{L}\right) + \cos 1.7207\frac{x}{L}$$

or

$$\frac{M}{Pe} = 1.162\left(\sin 1.721\frac{x}{L}\right) + \cos 1.721\frac{x}{L} \longleftarrow$$

(Note: kL and kx are in radians)

Bending-moment diagram for $P = 0.3\ P_{cr}$



Problem 11.5-4 Plot the load-deflection diagram for a pinned-end column with eccentric axial loads (see figure) if the eccentricity e of the load is 5 mm and the column has length $L = 3.6$ m, moment of inertia $I = 9.0 \times 10^6\ \text{mm}^4$, and modulus of elasticity $E = 210$ GPa.

Note: Plot the axial load as ordinate and the deflection at the midpoint as abscissa.

Solution 11.5-4 Column with eccentric loads

Column has pinned ends.

Use Eq. (11-54) for the deflection at the midpoint (maximum deflection):

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad (1)$$

Data

$e = 5.0$ mm $L = 3.6$ m $E = 210$ GPa

$I = 9.0 \times 10^6\ \text{mm}^4$

Critical load

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 1439.3\ \text{kN}$$

MAXIMUM DEFLECTION (FROM EQ. 1)

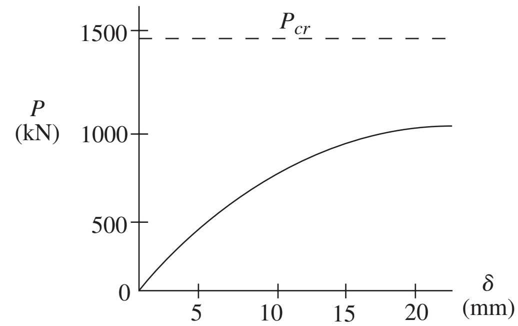
$$\delta = (5.0)[\sec(0.041404\sqrt{P}) - 1] \quad (2)$$

Units: P = kN δ = mm
angles are in radians.

SOLVE EQ. (2) FOR P:

$$P = 583.3\left[\arccos\left(\frac{5.0}{5.0+\delta}\right)\right]^2 \quad \longleftarrow$$

LOAD-DEFLECTION DIAGRAM



Problem 11.5-5 Solve the preceding problem for a column with $e = 0.20$ in., $L = 12$ ft, $I = 21.7$ in.^{4}, and $E = 30 \times 10^6$ psi.

Solution 11.5-5 Column with eccentric loads

Column has pinned ends
Use Eq. (11-54) for the deflection at the midpoint (maximum deflection):

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \quad (1)$$

DATA

$e = 0.20$ in. $L = 12$ ft $= 144$ in.
$E = 30 \times 10^6$ psi
$I = 21.7$ in.^{4}

CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 309.9\,k$$

MAXIMUM DEFLECTION (FROM EQ. 1)

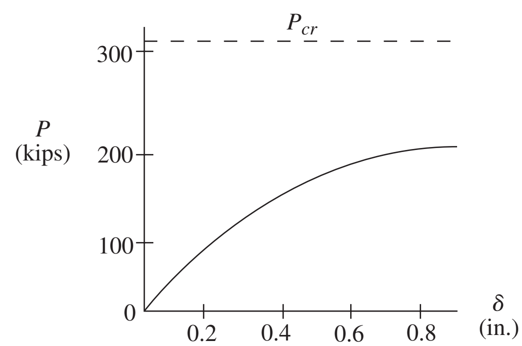
$$\delta = (0.20)[\sec(0.08924\sqrt{P}) - 1] \quad (2)$$

Units: P = kips δ = inches
Angles are in radians.

SOLVE EQ. (2) FOR P:

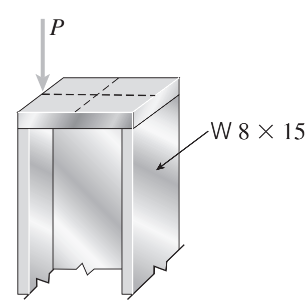
$$P = 125.6\left[\arccos\left(\frac{0.2}{0.2+\delta}\right)\right]^2 \quad \longleftarrow$$

LOAD-DEFLECTION DIAGRAM



Problem 11.5-6 A wide-flange member (W 8 × 15) is compressed by axial loads that have a resultant P acting at the point shown in the figure. The member has modulus of elasticity $E = 29{,}000$ ksi and pinned conditions at the ends. Lateral supports prevent any bending about the weak axis of the cross section.

If the length of the member is 20 ft and the deflection is limited to 1/4 inch, what is the maximum allowable load P_{allow}?



Solution 11.5-6 Column with eccentric axial load

Wide-flange member: W 8 × 15
$E = 29{,}000$ psi $L = 20$ ft $= 240$ in.
Maximum allowable deflection = 0.25 in. ($= \delta$)
Pinned-end conditions
Bending occurs about the strong axis (axis 1-1)

From Table E-1: $I = 48.0$ in.^{4}

$$e = \frac{8.11\text{ in.}}{2} = 4.055\text{ in.}$$

CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 238{,}500\text{ lb}$$

MAXIMUM DEFLECTION (EQ. 11-54)

$$\delta_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$0.25\text{ in.} = (4.055\text{ in.})[\sec(0.003216\sqrt{P}) - 1]$$

Rearrange terms and simplify:

$$\cos(0.003216\sqrt{P}) = 0.9419$$

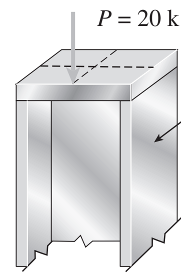
$$0.003216\sqrt{P} = \arccos 0.9419 = 0.3426$$

(Note: Angles are in radians)
Solve for P: $P = 11{,}300$ lb

ALLOWABLE LOAD

$P_{allow} = 11{,}300$ lb ←

Problem 11.5-7 A wide-flange member (W 10 × 30) is compressed by axial loads that have a resultant $P = 20$ k acting at the point shown in the figure. The material is steel with modulus of elasticity $E = 29{,}000$ ksi. Assuming pinned-end conditions, determine the maximum permissible length L_{max} if the deflection is not to exceed 1/400th of the length.



Solution 11.5-7 Column with eccentric axial load

Wide-flange member: W 10 × 30
Pinned-end conditions.
Bending occurs about the weak axis (axis 2-2).
$P = 20$ k $E = 29{,}000$ ksi L = length (inches)

Maximum allowable deflection $= \frac{L}{400}$ ($= \delta$)

From Table E-1: $I = 16.7$ in.^{4}

$$e = \frac{5.810\text{ in.}}{2} = 2.905\text{ in.}$$

$$k = \sqrt{\frac{P}{EI}} = 0.006426\text{ in.}^{-1}$$

DEFLECTION AT MIDPOINT (EQ. 11-51)

$$\delta = e\left(\sec\frac{kL}{2} - 1\right)$$

$$\frac{L}{400} = (2.905\text{ in.})[\sec(0.003213\,L) - 1]$$

Rearrange terms and simplify:

$$\sec(0.003213\,L) - 1 - \frac{L}{1162\text{ in.}} = 0$$

(Note: angles are in radians)
Solve the equation numerically for the length L:
$L = 150.5$ in.

MAXIMUM ALLOWABLE LENGTH

$L_{max} = 150.5$ in. $= 12.5$ ft ←

Problem 11.5-8 Solve the preceding problem (W 10 × 30) if the resultant force P equals 25 k.

Solution 11.5-8 Column with eccentric axial load

Wide-flange member: W 10 × 30
Pinned-end conditions
Bending occurs about the weak axis (axis 2-2)
$P = 25$ k $\quad E = 29{,}000$ ksi $\quad L =$ length (inches)

$$\text{Maximum allowable deflection} = \frac{L}{400} \; (= \delta)$$

From Table E-1: $I = 16.7 \text{ in.}^4$

$$e = \frac{5.810 \text{ in.}}{2} = 2.905 \text{ in.}$$

$$k = \sqrt{\frac{P}{EI}} = 0.007185 \text{ in.}^{-1}$$

DEFLECTION AT MIDPOINT (EQ. 11-51)

$$\delta = e\left(\sec\frac{kL}{2} - 1\right)$$

$$\frac{L}{400} = (2.905 \text{ in.})\,[\sec(0.003592\,L) - 1]$$

Rearrange terms and simplify:

$$\sec(0.003592\,L) - 1 - \frac{L}{1162 \text{ in.}} = 0$$

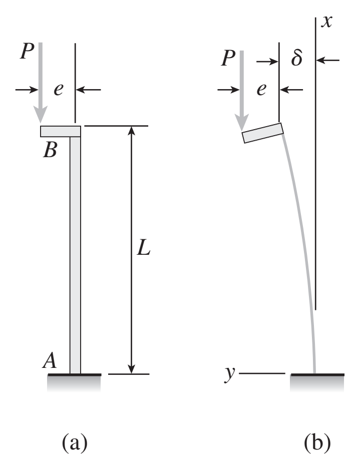
(Note: angles are in radians)
Solve the equation numerically for the length L:
$L = 122.6$ in.

MAXIMUM ALLOWABLE LENGTH

$L_{max} = 122.6 \text{ in.} = 10.2 \text{ ft}$ ←

Problem 11.5-9 The column shown in the figure is fixed at the base and free at the upper end. A compressive load P acts at the top of the column with an eccentricity e from the axis of the column.

Beginning with the differential equation of the deflection curve, derive formulas for the maximum deflection δ of the column and the maximum bending moment M_{max} in the column.



Solution 11.5-9 Fixed-free column

e = eccentricity of load P
δ = deflection at the end of the column
v = deflection of the column at distance x from the base

DIFFERENTIAL EQUATION (EQ. 11.3)

$$EIv'' = M = P(e + \delta - v) \qquad k^2 = \frac{P}{EI}$$

$$v'' = k^2(e + \delta - v)$$

$$v'' + k^2 v = k^2(e + \delta)$$

GENERAL SOLUTION

$$v = C_1 \sin kx + C_2 \cos kx + e + \delta$$

$$v' = C_1 k \cos kx - C_2 k \sin kx$$

B.C. 1 $\quad v(0) = 0 \quad \therefore C_2 = -e - \delta$
B.C. 2 $\quad v'(0) = 0 \quad \therefore C_1 = 0$
$v = (e + \delta)(1 - \cos kx)$
B.C. 3 $\quad v(L) = \delta \quad \therefore \delta = (e + \delta)(1 - \cos kL)$
or $\quad \delta = e(\sec kL - 1)$

MAXIMUM DEFLECTION $\quad \delta = e(\sec kL - 1)$ ←

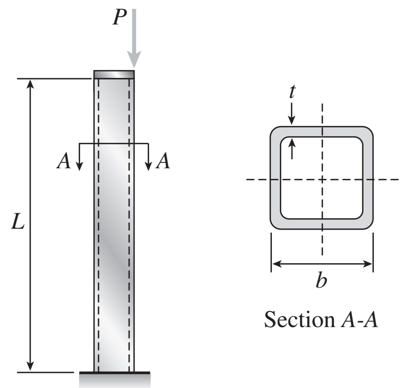
MAXIMUM LENDING MOMENT (AT BASE OF COLUMN)

$M_{max} = P(e + \delta) = Pe \sec kL$ ←

NOTE: $\quad v = (e + \delta)(1 - \cos kx)$
$\quad = e(\sec kL)(1 - \cos kx)$

Problem 11.5-10 An aluminum box column of square cross section is fixed at the base and free at the top (see figure). The outside dimension b of each side is 100 mm and the thickness t of the wall is 8 mm. The resultant of the compressive loads acting on the top of the column is a force $P = 50$ kN acting at the outer edge of the column at the midpoint of one side.

What is the longest permissible length L_{max} of the column if the deflection at the top is not to exceed 30 mm? (Assume $E = 73$ GPa.)



Probs. 11.5-10 and 11.5-11

Solution 11.5-10 Fixed-free column

δ = deflection at the top

Use Eq. (11-51) with $L/2$ replaced by L:

$\delta = e(\sec kL - 1)$ (1)

(This same equation is obtained in Prob. 11.5-9.)

SOLVE FOR L FROM EQ. (1)

$$\sec kL = 1 + \frac{\delta}{e} = \frac{e+\delta}{e}$$

$$\cos kL = \frac{e}{e+\delta} \qquad kL = \arccos\frac{e}{e+\delta}$$

$$L = \frac{1}{k}\arccos\frac{e}{e+\delta} \qquad k = \sqrt{\frac{P}{EI}}$$

$$L = \sqrt{\frac{EI}{P}}\arccos\frac{e}{e+\delta} \qquad (2)$$

NUMERICAL DATA

$E = 73$ GPa $\quad b = 100$ mm $\quad t = 8$ mm

$P = 50$ kN $\quad \delta = 30$ mm $\quad e = \dfrac{b}{2} = 50\,\text{mm}$

$$I = \frac{1}{12}[b^4 - (b-2t)^4] = 4.1844 \times 10^6\,\text{mm}^4$$

MAXIMUM ALLOWABLE LENGTH

Substitute numerical data into Eq. (2).

$$\sqrt{\frac{EI}{P}} = 2.4717\,\text{m} \qquad \frac{e}{e+\delta} = 0.625$$

$$\arccos\frac{e}{e+\delta} = 0.89566\,\text{radians}$$

$L_{max} = (2.4717\text{ m})(0.89566) = 2.21$ m ⟵

Problem 11.5-11 Solve the preceding problem for an aluminum column with $b = 6.0$ in., $t = 0.5$ in., $P = 30$ k, and $E = 10.6 \times 10^3$ ksi. The deflection at the top is limited to 2.0 in.

Solution 11.5-11 Fixed-free column

δ = deflection at the top

Use Eq. (11-51) with $L/2$ replaced by L:

$\delta = e(\sec kL - 1)$ (1)

(This same equation is obtained in Prob. 11.5-9.)

SOLVE FOR L FROM EQ. (1)

$$\sec kL = 1 + \frac{\delta}{e} = \frac{e+\delta}{e}$$

$$\cos kL = \frac{e}{e+\delta} \qquad kL = \arccos\frac{e}{e+\delta}$$

$$L = \frac{1}{k}\arccos\frac{e}{e+\delta} \qquad k = \sqrt{\frac{P}{EI}}$$

$$L = \sqrt{\frac{EI}{P}}\arccos\frac{e}{e+\delta} \qquad (2)$$

NUMERICAL DATA

$E = 10.6 \times 10^3$ ksi $\quad b = 6.0$ in. $\quad t = 0.5$ in.

$P = 30$ k $\quad \delta = 2.0$ in. $\quad e = \dfrac{b}{2} = 3.0\,\text{in.}$

$$I = \frac{1}{12}[b^4 - (b-2t)^4] = 55.917\,\text{in.}^4$$

MAXIMUM ALLOWABLE LENGTH

Substitute numerical data into Eq. (2).

$$\sqrt{\frac{EI}{P}} = 140.56\,\text{in.} \qquad \frac{e}{e+\delta} = 0.60$$

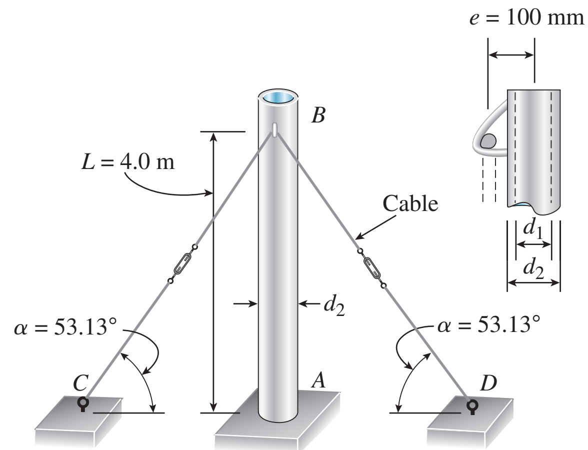
$$\arccos\frac{e}{e+\delta} = 0.92730\,\text{radians}$$

$$L_{max} = (140.56\text{ in.})(0.92730)$$
$$= 130.3\text{ in.} = 10.9\text{ ft} \quad \longleftarrow$$

Problem 11.5-12 A steel post *AB* of hollow circular cross section is fixed at the base and free at the top (see figure). The inner and outer diameters are $d_1 = 96$ mm and $d_2 = 110$ mm, respectively, and the length $L = 4.0$ m.

A cable *CBD* passes through a fitting that is welded to the side of the post. The distance between the plane of the cable (plane *CBD*) and the axis of the post is $e = 100$ mm, and the angles between the cable and the ground are $\alpha = 53.13°$. The cable is pretensioned by tightening the turnbuckles.

If the deflection at the top of the post is limited to $\delta = 20$ mm, what is the maximum allowable tensile force *T* in the cable? (Assume $E = 205$ GPa.)



Solution 11.5-12 **Fixed-free column**

δ = deflection at the top

P = compressive force in post $\qquad k = \sqrt{\frac{P}{EI}}$

Use Eq. (11-51) with $L/2$ replaced by L:

$$\delta = e(\sec kL - 1) \qquad (1)$$

(This same equation in obtained in Prob. 11.5-9.)

SOLVE FOR P FROM EQ.(1)

$$\sec kL = 1 + \frac{\delta}{e} = \frac{e+\delta}{e}$$

$$\cos kL = \frac{e}{e+\delta} \qquad kL = \arccos\frac{e}{e+\delta}$$

$$kL = \sqrt{\frac{PL^2}{EI}} \qquad \sqrt{\frac{PL^2}{EI}} = \arccos\frac{e}{e+\delta}$$

Square both sides and solve for P:

$$P = \frac{EI}{L^2}\left(\arccos\frac{e}{e+\delta}\right)^2 \qquad (2)$$

NUMERICAL DATA

$E = 205$ GPa $\quad L = 4.0$ m $\quad e = 100$ mm
$\delta = 20$ mm $\quad d_2 = 110$ mm $\quad d_1 = 96$ mm

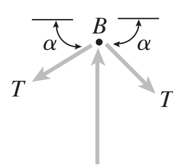
$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.0177 \times 10^6\,\text{mm}^4$$

MAXIMUM ALLOWABLE COMPRESSIVE FORCE P

Substitute numerical data into Eq. (2).

$$P_{allow} = 13{,}263\text{ N} = 13{,}263\text{ kN}$$

MAXIMUM ALLOWABLE TENSILE FORCE T IN THE CABLE



Free-body diagram of joint *B*:

$\alpha = 53.13°$

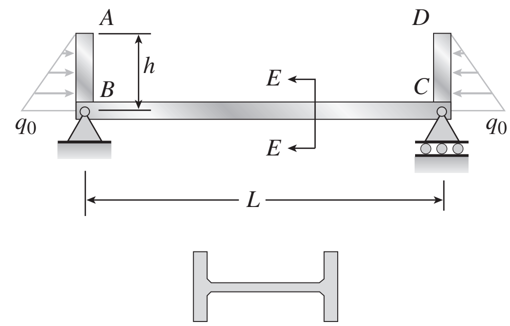
$$\sum F_{vert} = 0 \qquad P - 2T\sin\alpha = 0$$

$$T = \frac{P}{2\sin\alpha} = \frac{5P}{8} = 8289\,\text{N}$$

Problem 11.5-13 A frame *ABCD* is constructed of steel wide-flange members (W 8 × 21; $E = 30 \times 10^6$ psi) and subjected to triangularly distributed loads of maximum intensity q_0 acting along the vertical members (see figure). The distance between supports is $L = 20$ ft and the height of the frame is $h = 4$ ft. The members are rigidly connected at *B* and *C*.

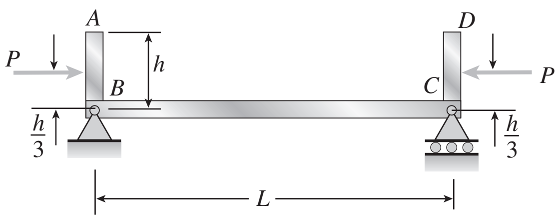
(a) Calculate the intensity of load q_0 required to produce a maximum bending moment of 80 k-in. in the horizontal member *BC*.

(b) If the load q_0 is reduced to one-half of the value calculated in part (a), what is the maximum bending moment in member *BC*? What is the ratio of this moment to the moment of 80 k-in. in part (a)?



Section *E*-*E*

Solution 11.5-13 **Frame with triangular loads**



P = resultant force
e = eccentricity

$$P = \frac{q_0 h}{2} \qquad e = \frac{h}{3}$$

Maximum bending moment in beam *BC*

From Eq. (11-56): $\quad M_{max} = Pe\sec\frac{kL}{2}$

$$k = \sqrt{\frac{P}{EI}} \qquad \therefore\ M_{max} = Pe\sec\sqrt{\frac{PL^2}{4EI}} \qquad (1)$$

Numerical data

W 8 × 21 $\quad I = I_2 = 9.77\text{ in.}^4$ (from Table E-1)
$E = 30 \times 10^6$ psi $\quad L = 20$ ft = 240 in.
$h = 4$ ft = 48 in.

$$e = \frac{h}{3} = 16\,\text{in.}$$

(a) Load q_0 to produce M_{max} = 80 k-in.

Substitute numerical values into Eq. (1).
Units: pounds and inches

$$M_{max} = 80{,}000\,\text{lb-in.}\sqrt{\frac{PL^2}{4EI}}$$

$$= 0.0070093\sqrt{P} \qquad \text{(radians)}$$

$$80{,}000 = P(16\,\text{in.})[\sec(0.0070093\sqrt{P})]$$

$$5{,}000 = P\sec(0.0070093\sqrt{P})$$

$$P - 5{,}000[\cos(0.0070093\sqrt{P})] = 0 \qquad (2)$$

Solve Eq. (2) numerically

$P = 4461.9$ lb

$$q_0 = \frac{2P}{h} = 186\,\text{lb/in.} = 2230\,\text{lb/ft} \longleftarrow$$

(b) Load q_0 is reduced to one-half its value

∴ P is reduced to one-half its value.

$$P = \frac{1}{2}(4461.9\,\text{lb}) = 2231.0\,\text{lb}$$

Substitute numerical values into Eq. (1) and solve for M_{max}.

M_{max} = 37.75 k-in. ⟵

Ratio: $\frac{M_{max}}{80\,\text{k-in.}}\ 5\ \frac{37.7}{80}\ 5\ 0.47$ ⟵

This result shows that the bending moment varies nonlinearly with the load.

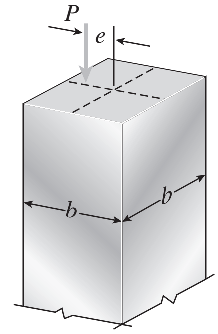
The Secant Formula

When solving the problems for Section 11.6, assume that bending occurs in the principal plane containing the eccentric axial load.

Problem 11.6-1 A steel bar has a square cross section of width $b = 2.0$ in. (see figure). The bar has pinned supports at the ends and is 3.0 ft long. The axial forces acting at the end of the bar have a resultant $P = 20$ k located at distance $e = 0.75$ in. from the center of the cross section. Also, the modulus of elasticity of the steel is 29,000 ksi.

(a) Determine the maximum compressive stress σ_{max} in the bar.

(b) If the allowable stress in the steel is 18,000 psi, what is the maximum permissible length L_{max} of the bar?



Probs. 11.6-1 through 11.6-3

Solution 11.6-1 **Bar with square cross section**

Pinned supports.

DATA

$b = 2.0$ in. $L = 3.0$ ft $= 36$ in. $P = 20$ k
$e = 0.75$ in. $E = 29{,}000$ ksi

(a) MAXIMUM COMPRESSIVE STRESS

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$$\frac{P}{A} = \frac{P}{b^2} = 5.0\,\text{ksi} \qquad c = \frac{b}{2} = 1.0\,\text{in.}$$

$$I = \frac{b^4}{12} = 1.333\,\text{in.}^4 \qquad r^2 = \frac{I}{A} = 0.3333\,\text{in.}^2$$

$$\frac{ec}{r^2} = 2.25 \qquad \frac{L}{r} = 62.354 \qquad \frac{P}{EA} = 0.00017241$$

Substitute into Eq. (1):

$\sigma_{max} = 17.3$ ksi ←

(b) MAXIMUM PERMISSIBLE LENGTH

$\sigma_{allow} = 18{,}000$ psi

Solve Eq. (1) for the length L:

$$L = 2\sqrt{\frac{EI}{P}}\arccos\left[\frac{P(ec/r^2)}{\sigma_{max}A - P}\right] \quad (2)$$

Substitute numerical values:

$L_{max} = 46.2$ in. ←

Problem 11.6-2 A brass bar ($E = 100$ GPa) with a square cross section is subjected to axial forces having a resultant P acting at distance e from the center (see figure). The bar is pin supported at the ends and is 0.6 m in length. The side dimension b of the bar is 30 mm and the eccentricity e of the load is 10 mm.

If the allowable stress in the brass is 150 MPa, what is the allowable axial force P_{allow}?

Solution 11.6-2 **Bar with square cross section**

Pinned supports.

DATA

$b = 30$ mm $L = 0.6$ m $\sigma_{allow} = 150$ MPa
$e = 10$ mm $E = 100$ GPa

SECANT FORMULA (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

Units: Newtons and meters

$\sigma_{max} = 150 \times 10^6\ \text{N/m}^2$

$A = b^2 = 900 \times 10^{-6}\ \text{m}^2$

$$c = \frac{b}{2} = 0.015\ \text{m} \qquad r^2 = \frac{I}{A} = \frac{b^2}{12} = 75 \times 10^{-6}\ \text{m}^2$$

$$\frac{ec}{r^2} = 2.0 \qquad P = \text{newtons} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.0036515\sqrt{P}$$

Substitute numerical values into Eq. (1):

$$150 \times 10^6 = \frac{P}{900 \times 10^{-6}}\,[1 + 2\sec(0.0036515\sqrt{P})]$$

or

$$P[1 + 2\sec(0.0036515\sqrt{P})] - 135{,}000 = 0 \qquad (2)$$

Solve Eq. (2) numerically:

$P_{allow} = 37{,}200 \quad N = 37.2$ kN ←

Problem 11.6-3 A square aluminum bar with pinned ends carries a load $P = 25$ k acting at distance $e = 2.0$ in. from the center (see figure on the previous page). The bar has length $L = 54$ in. and modulus of elasticity $E = 10{,}600$ ksi.

If the stress in the bar is not to exceed 6 ksi, what is the minimum permissible width b_{min} of the bar?

Solution 11.6-3 Square aluminum bar

Pinned ends

Data

Units: pounds and inches

$P = 25\ \text{k} = 25{,}000\ \text{psi} \qquad e = 2.0\ \text{in.}$

$L = 54\ \text{in.} \qquad E = 10{,}600\ \text{ksi} = 10{,}600{,}000\ \text{psi}$

$\sigma_{max} = 6.0\ \text{ksi} = 6{,}000\ \text{psi}$

Secant formula (Eq. 11-59)

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$$A = b^2 \qquad c = \frac{b}{2} \qquad r^2 = \frac{I}{A} = \frac{b^2}{12}$$

$$\frac{ec}{r^2} = \frac{12}{b} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = \frac{4.5423}{b^2}$$

Substitute terms into Eq. (1):

$$6{,}000 = \frac{25{,}000}{b^2}\left[1 + \frac{12}{b}\sec\left(\frac{4.5423}{b^2}\right)\right]$$

or

$$1 + \frac{12}{b}\sec\left(\frac{4.5423}{b^2}\right) - 0.24\,b^2 = 0 \qquad (2)$$

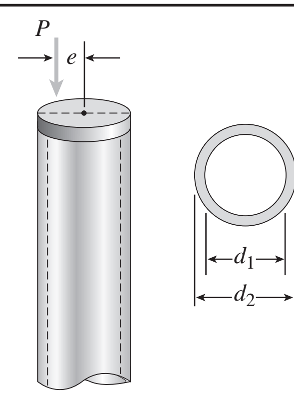
Solve Eq. (2) numerically:

$b_{min} = 4.10$ in. ←

Problem 11.6-4 A pinned-end column of length $L = 2.1$ m is constructed of steel pipe ($E = 210$ GPa) having inside diameter $d_1 = 60$ mm and outside diameter $d_2 = 68$ mm (see figure). A compressive load $P = 10$ kN acts with eccentricity $e = 30$ mm.

(a) What is the maximum compressive stress σ_{max} in the column?

(b) If the allowable stress in the steel is 50 MPa, what is the maximum permissible length L_{max} of the column?



Probs. 11.6-4 through 11.6-6

Solution 11.6-4 Steel pipe column

Pinned ends.

DATA Units: Newtons and meters

$L = 2.1$ m $E = 210$ GPa $= 210 \times 10^9$ N/m^2
$d_1 = 60$ mm $= 0.06$ m $d_2 = 68$ mm $= 0.068$ m
$P = 10$ kN $= 10{,}000$ N $e = 30$ mm $= 0.03$ m

TUBULAR CROSS SECTION

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 804.25 \times 10^{-6}\text{m}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 413.38 \times 10^{-9}\text{m}^4$$

(a) MAXIMUM COMPRESSIVE STRESS

Secant formula (Eq. 11-59):

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$$\frac{P}{A} = 12.434 \times 10^6\,\text{N/m}^2$$

$$r^2 = \frac{I}{A} = 513.99 \times 10^{-6}\,m^2$$

$$r = 22.671 \times 10^{-3}\,\text{m} \qquad c = \frac{d_2}{2} = 0.034\,\text{m}$$

$$\frac{ec}{r^2} = 1.9845 \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.35638$$

Substitute into Eq. (1):

$\sigma_{\max} = 38.8 \times 10^6$ N/m^2 $= 38.8$ MPa ←

(b) MAXIMUM PERMISSIBLE LENGTH

$\sigma_{\text{allow}} = 50$ MPa

Solve Eq. (1) for the length L:

$$L = 2\sqrt{\frac{EI}{P}}\arccos\left[\frac{P(ec/r^2)}{\sigma_{\max}A - P}\right] \qquad (2)$$

Substitute numerical values:

$L_{\max} = 5.03$ m ←

Problem 11.6-5 A pinned-end strut of length $L = 5.2$ ft is constructed of steel pipe ($E = 30 \times 10^3$ ksi) having inside diameter $d_1 = 2.0$ in. and outside diameter $d_2 = 2.2$ in. (see figure). A compressive load $P = 2.0$ k is applied with eccentricity $e = 1.0$ in.

(a) What is the maximum compressive stress $\sigma_{\max}$ in the strut?

(b) What is the allowable load P_{allow} if a factor of safety $n = 2$ with respect to yielding is required? (Assume that the yield stress σ_Y of the steel is 42 ksi.)

Solution 11.6-5 Pinned-end strut

Steel pipe.

DATA Units: kips and inches

$L = 5.2$ ft $= 62.4$ in. $E = 30 \times 10^3$ ksi
$d_1 = 2.0$ in. $d_2 = 2.2$ in.
$P = 2.0$ k $e = 1.0$ in.

TUBULAR CROSS SECTION

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 0.65973\,\text{in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 0.36450\,\text{in.}^4$$

(a) MAXIMUM COMPRESSIVE STRESS

Secant formula (Eq. 11-59):

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$$\frac{P}{A} = 3.0315\,\text{ksi} \qquad c = \frac{d_2}{2} = 1.1\,\text{in.}$$

$$r^2 = \frac{I}{A} = 0.55250\,\text{in.}^2 \qquad \frac{ec}{r^2} = 1.9910$$

$$r = 0.74330\,\text{in.} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.42195$$

Substitute into Eq. (1):

$\sigma_{\max} = 9.65$ ksi ←

(b) ALLOWABLE LOAD

$\sigma_Y = 42$ ksi $n = 2$ find P_{allow}

Substitute numerical values into Eq. (1):

$$42 = \frac{P}{0.65973}[1 + 1.9910\sec(0.29836\sqrt{P})] \qquad (2)$$

Solve Eq. (2) numerically: $P = P_Y = 7.184$ k

$$P_{\text{allow}} = \frac{P_Y}{n} = 3.59\,\text{k}$$ ←

Problem 11.6-6 A circular aluminum tube with pinned ends supports a load $P = 18$ kN acting at distance $e = 50$ mm from the center (see figure). The length of the tube is 3.5 m and its modulus of elasticity is 73 GPa.

If the maximum permissible stress in the tube is 20 MPa, what is the required outer diameter d_2 if the ratio of diameters is to be $d_1/d_2 = 0.9$?

Solution 11.6-6 Aluminum tube

Pinned ends.

DATA $P = 18$ kN $e = 50$ mm

$L = 3.5$ m $E = 73$ GPa

$\sigma_{max} = 20$ MPa $d_1/d_2 = 0.9$

SECANT FORMULA (EQ. 11-59)

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[d_2^2 - (0.9\,d_2)^2] = 0.14923\,d_2^2$$

(d_2 = mm; A = mm^2)

$$\frac{P}{A} = \frac{18{,}000\,\text{N}}{0.14923\,d_2^2} = \frac{120{,}620}{d_2^2}\left(\frac{P}{A} = \text{MPa}\right)$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = \frac{\pi}{64}[d_2^4 - (0.9\,d_2)^4] = 0.016881\,d_2^4$$

(d_2 = mm; I = mm^4)

$$r^2 = \frac{I}{A} = 0.11313\,d_2^2 \quad (d_2 = \text{mm};\ r^2 = \text{mm}^2)$$

$r = 0.33634\,d_2$ (r = mm)

$$c = \frac{d_2}{2} \qquad \frac{ec}{r^2} = \frac{(50\,\text{mm})(d_2/2)}{0.11313\,d_2^2} = \frac{220.99}{d_2}$$

$$\frac{L}{2r} = \frac{3500\,\text{mm}}{2(0.33634\,d_2)} = \frac{5{,}203.1}{d_2}$$

$$\frac{P}{EA} = \frac{18{,}000\,\text{N}}{(73{,}000\,\text{N/mm}^2)(0.14923\,d_2^2)} = \frac{1.6524}{d_2^2}$$

$$\frac{L}{2r}\sqrt{\frac{P}{EA}} = \frac{5{,}203.1}{d_2}\sqrt{\frac{1.6524}{d_2^2}} = \frac{6688.2}{d_2^2}$$

SUBSTITUTE THE ABOVE EXPRESSIONS INTO EQ. (1):

$$\sigma_{max} = 20\,\text{MPa} = \frac{120{,}620}{d_2^2} + \left[1 + \frac{220.99}{d_2}\sec\left(\frac{6688.2}{d_2^2}\right)\right] \quad (2)$$

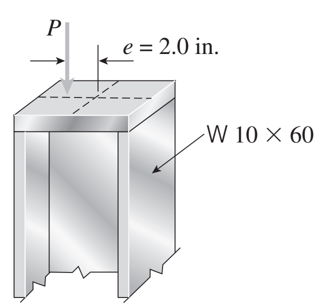
SOLVE EQ. (2) NUMERICALLY:

$d_2 = 131$ mm ←

Problem 11.6-7 A steel column ($E = 30 \times 10^3$ ksi) with pinned ends is constructed of a W 10 × 60 wide-flange shape (see figure). The column is 24 ft long. The resultant of the axial loads acting on the column is a force P acting with an eccentricity $e = 2.0$ in.

(a) If $P = 120$ k, determine the maximum compressive stress σ_{max} in the column.

(b) Determine the allowable load P_{allow} if the yield stress is $\sigma_Y = 42$ ksi and the factor of safety with respect to yielding of the material is $n = 2.5$.



Solution 11.6-7 **Steel column with pinned ends**

$E = 30 \times 10^3$ ksi $\quad L = 24$ ft $= 288$ in.
$e = 2.0$ in.
W 10×60 wide-flange shape
$A = 17.6$ in.^{2} $\quad I = 341$ in.^{4} $\quad d = 10.22$ in.

$$r^2 = \frac{I}{A} = 19.38 \text{ in.}^2 \quad r = 4.402 \text{ in.} \quad c = \frac{d}{2} = 5.11 \text{ in.}$$

$$\frac{L}{r} = 65.42 \qquad \frac{ec}{r^2} = 0.5273$$

(a) Maximum compressive stress ($P = 120$ k)

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$$\frac{P}{A} = 6.818 \text{ ksi} \quad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.4931$$

Substitute into Eq. (1): $\quad \sigma_{max} = 10.9$ ksi ←

(b) Allowable load

$\sigma_Y = 42$ ksi $\quad n = 2.5 \quad$ find P_{allow}
Substitute into Eq. (1):

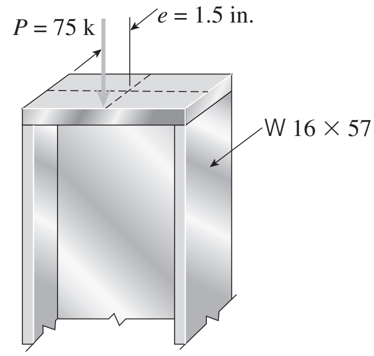
$$42 = \frac{P}{17.6}[1 + 0.5273\sec(0.04502\sqrt{P})]$$

Solve numerically: $P = P_Y = 399.9$ k
$P_{allow} = P_Y/n = 160$ k ←

Problem 11.6-8 A W 16×57 steel column is compressed by a force $P = 75$ k acting with an eccentricity $e = 1.5$ in., as shown in the figure. The column has pinned ends and length L. Also, the steel has modulus of elasticity $E = 30 \times 10^3$ ksi and yield stress $\sigma_Y = 36$ ksi.

(a) If the length $L = 10$ ft, what is the maximum compressive stress σ_{max} in the column?

(b) If a factor of safety $n = 2.0$ is required with respect to yielding, what is the longest permissible length L_{max} of the column?



Solution 11.6-8 **Steel column with pinned ends**

W 16×57 $\quad A = 16.8$ in.^{2} $\quad I = I_2 = 43.1$ in.^{4}
$b = 7.120$ in.
$c = b/2 = 3.560$ in.

$$e = 1.5 \text{ in.} \qquad r^2 = \frac{I}{A} = 2.565 \text{ in.}^2$$

$$\frac{ec}{r^2} = 2.082 \qquad r = 1.602 \text{ in.}$$

$$P = 75 \text{ k} \quad E = 30 \times 10^3 \text{ ksi} \quad \frac{P}{EA} = 148.8 \times 10^{-6}$$

(a) Maximum compressive stress

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$L = 10$ ft $= 120$ in.

$$\frac{P}{A} = 4.464 \text{ ksi} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.4569$$

Substitute into Eq. (1):

$$\sigma_{max} = 4.464\,[1 + 2.082\sec(0.4569)]$$
$$= 14.8 \text{ ksi} \leftarrow$$

(b) Maximum length

Solve Eq. (1) for the length L:

$$L = 2\sqrt{\frac{EI}{P}}\arccos\left[\frac{P(ec/r^2)}{\sigma_{max}A - P}\right] \qquad (2)$$

$\sigma_Y = 36$ ksi $\quad n = 2.0 \quad P_Y = n \quad P = 150$ k

Substitute P_Y for P and σ_Y for σ_{max} in Eq. (2):

$$L_{max} = 2\sqrt{\frac{EI}{P_Y}}\arccos \text{B}\frac{P_Y(ec/r^2)}{\sigma_Y A - P_Y}\Big] \qquad (3)$$

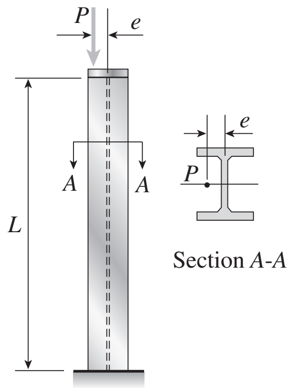
Substitute numerical values in Eq. (3) and solve for L_{max}:

$L_{max} = 151.1$ in. $= 12.6$ ft ←

Problem 11.6-9 A steel column ($E = 30 \times 10^3$ ksi) that is fixed at the base and free at the top is constructed of a W 8 × 35 wide-flange member (see figure). The column is 9.0 ft long. The force P acting at the top of the column has an eccentricity $e = 1.25$ in.

(a) If $P = 40$ k, what is the maximum compressive stress in the column?

(b) If the yield stress is 36 ksi and the required factor of safety with respect to yielding is 2.1, what is the allowable load P_{allow}?



Probs. 11.6-9 and 11.6-10

Solution 11.6-9 **Steel column (fixed-free)**

$E = 30 \times 10^3$ ksi $\quad e = 1.25$ in.

$Le = 2L = 2\,(9.0 \text{ ft}) = 18 \text{ ft} = 216$ in.

W 8 × 35 WIDE-FLANGE SHAPE

$A = 10.3 \text{ in.}^2 \quad I = I_2 = 42.6 \text{ in.}^4 \quad b = 8.020$ in.

$$r^2 = \frac{I}{A} = 4.136 \text{ in.}^2 \quad r = 2.034 \text{ in.}$$

$$c = \frac{b}{2} = 4.010 \text{ in.} \quad \frac{Le}{r} = 106.2 \quad \frac{ec}{r^2} = 1.212$$

(a) MAXIMUM COMPRESSIVE STRESS ($P = 40$ k)

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{Le}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$$\frac{P}{A} = 3.883 \text{ ksi} \quad \frac{Le}{2r}\sqrt{\frac{P}{EA}} = 0.6042$$

Substitute into Eq. (1): $\quad \sigma_{max} = 9.60$ ksi ←

(b) ALLOWABLE LOAD

$\sigma_Y = 36$ ksi $\quad n = 2.1 \quad$ find P_{allow}

Substitute into Eq. (1):

$$36 = \frac{P}{10.3}[1 + 1.212 \sec(0.09552\sqrt{P})]$$

Solve numerically: $\quad P = P_Y = 112.6$ k

$P_{allow} = P_Y/n = 53.6$ k ←

Problem 11.6-10 A W 12 × 50 wide-flange steel column with length $L = 12.5$ ft is fixed at the base and free at the top (see figure). The load P acting on the column is intended to be centrally applied, but because of unavoidable discrepancies in construction, an eccentricity ratio of 0.25 is specified. Also, the following data are supplied: $E = 30 \times 10^3$ ksi, $\sigma_Y = 42$ ksi, and $P = 70$ k.

(a) What is the maximum compressive stress σ_{max} in the column?

(b) What is the factor of safety n with respect to yielding of the steel?

Solution 11.6-10 **Steel column (fixed-free)**

$$E = 30 \times 10^3 \text{ ksi} \quad \frac{ec}{r^2} = 0.25$$

$Le = 2L = 2\,(12.5 \text{ ft}) = 25 \text{ ft} = 300$ in.

W 12 × 50 WIDE-FLANGE SHAPE

$A = 14.7 \text{ in.}^2 \quad I = I_2 = 56.3 \text{ in.}^4$

$$r^2 = \frac{I}{A} = 3.830 \text{ in.}^2 \quad r = 1.957 \text{ in.}$$

(a) MAXIMUM COMPRESSIVE STRESS ($P = 70$ k)

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

$$\frac{P}{A} = 4.762 \text{ ksi} \quad \frac{L_e}{2r}\sqrt{\frac{P}{EA}} = 0.9657$$

Substitute into Eq. (1): $\sigma_{max} = 6.85$ ksi ⟵

(b) Factor of safety with respect to yielding

$\sigma_Y = 42$ psi

Substitute into Eq. (1) with $\sigma_{max} = \sigma_Y$ and $P = P_Y$:

$$42 = \frac{P_Y}{A}[1 + 0.25\sec(0.1154\sqrt{P_Y})]$$

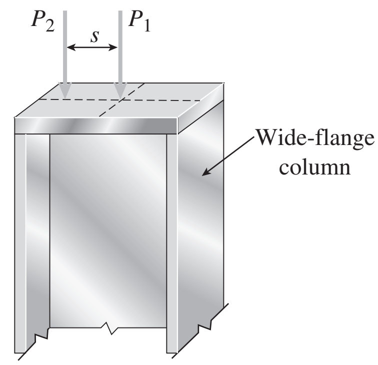
Solve numerically: $P_Y = 164.5$ k

$$P = 70\,\text{k} \quad n = \frac{P_Y}{P} = \frac{164.5\,\text{k}}{70\,\text{k}} = 2.35 \longleftarrow$$

Problem 11.6-11 A pinned-end column with length $L = 18$ ft is constructed from a W 12 × 87 wide-flange shape (see figure). The column is subjected to a centrally applied load $P_1 = 180$ k and an eccentrically applied load $P_2 = 75$ k. The load P_2 acts at distance $s = 5.0$ in. from the centroid of the cross section. The properties of the steel are $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.

(a) Calculate the maximum compressive stress in the column.

(b) Determine the factor of safety with respect to yielding.



Probs. 11.6.11 and 11.6.12

Solution 11.6-11 Column with two loads

Pinned-end column. W 12 × 87

Data

$L = 18$ ft $= 216$ in.

$P_1 = 180$ k $P_2 = 75$ k $s = 5.0$ in.

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

$$P = P_1 + P_2 = 255\,\text{k} \qquad e = \frac{P_2 s}{P} = 1.471\,\text{in.}$$

$A = 25.6$ in.^{2} $I = I_1 = 740$ in.^{4} $d = 12.53$ in.

$$r^2 = \frac{I}{A} = 28.91\,\text{in.}^2 \qquad r = 5.376\,\text{in.}$$

$$c = \frac{d}{2} = 6.265\,\text{in.} \qquad \frac{ec}{r^2} = 0.3188$$

$$\frac{P}{A} = 9.961\,\text{ksi} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.3723$$

(a) Maximum compressive stress

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

Substitute into Eq. (1): $\sigma_{max} = 13.4$ ksi ⟵

(b) Factor of safety with respect to yielding

$\sigma_{max} = \sigma_Y = 36$ ksi $P = P_Y$

Substitute into Eq. (1):

$$36 = \frac{P_Y}{25.6}[1 + 0.3188\sec(0.02332\sqrt{P_Y})]$$

Solve numerically: $P_Y = 664.7$ k

$$P = 2.55\,\text{k} \quad n = \frac{P_Y}{P} = \frac{664.7\,\text{k}}{255\,\text{k}} = 2.61 \longleftarrow$$

Problem 11.6-12 The wide-flange pinned-end column shown in the figure carries two loads, a force $P_1 = 100$ k acting at the centroid and a force $P_2 = 60$ k acting at distance $s = 4.0$ in. from the centroid. The column is a W 10 × 45 shape with $L = 13.5$ ft, $E = 29 \times 10^3$ ksi, and $\sigma_Y = 42$ ksi.

(a) What is the maximum compressive stress in the column?

(b) If the load P_1 remains at 100 k, what is the largest permissible value of the load P_2 in order to maintain a factor of safety of 2.0 with respect to yielding?

Solution 11.6-12 Column with two loads

Pinned-end column. W 10 × 45

DATA

$L = 13.5 \text{ ft} = 162 \text{ in.}$

$P_1 = 100 \text{ k} \quad P_2 = 60 \text{ k} \quad s = 4.0 \text{ in.}$

$E = 29{,}000 \text{ ksi} \quad \sigma_Y = 42 \text{ ksi}$

$$P = P_1 + P_2 = 160\,\text{k} \qquad e = \frac{P_2 s}{P} = 1.50\,\text{in.}$$

$A = 13.3 \text{ in.}^2 \quad I = I_1 = 248 \text{ in.}^4 \quad d = 10.10 \text{ in.}$

$$r^2 = \frac{I}{A} = 18.65\,\text{in.}^2 \qquad r = 4.318\,\text{in.}$$

$$c = \frac{d}{2} = 5.05\,\text{in.} \qquad \frac{ec}{r^2} = 0.4062$$

$$\frac{P}{A} = 12.03\,\text{ksi} \qquad \frac{L}{2r}\sqrt{\frac{P}{EA}} = 0.3821$$

(a) MAXIMUM COMPRESSIVE STRESS

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad (1)$$

Substitute into Eq. (1): $\sigma_{max} = 17.3$ ksi ⟵

(b) LARGEST VALUE OF LOAD P_2

$P_1 = 100$ k (no change)

$n = 2.0$ with respect to yielding

Units: kips, inches

$P = P_1 + P_2 = 100 + P_2$

$$e = \frac{P_2 s}{P} = \frac{P_2(4.0)}{100 + P_2} \qquad \frac{ec}{r^2} = \frac{1.0831\,P_2}{100 + P_2}$$

$\sigma_{max} = \sigma_Y = 42$ ksi $\quad P_Y = n\,P = 2.0\,(100 + P_2)$

Use Eq. (1) with σ_{max} replaced by σ_Y and P replaced by P_Y:

$$\sigma_Y = \frac{P_Y}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P_Y}{EA}}\right)\right] \qquad (2)$$

Substitute into Eq. (2):

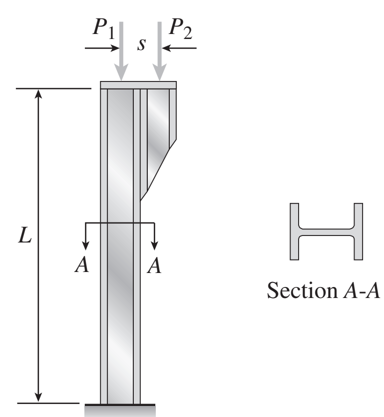
$$42 = \frac{2.0(100 + P_2)}{13.3}\left[1 + \frac{1.0831\,P_2}{100 + P_2}\sec\left(0.04272\sqrt{100 + P_2}\right)\right]$$

Solve numerically: $P_2 = 78.4$ k ⟵

Problem 11.6-13 A W 14 × 53 wide-flange column of length $L = 15$ ft is fixed at the base and free at the top (see figure). The column supports a centrally applied load $P_1 = 120$ k and a load $P_2 = 40$ k supported on a bracket. The distance from the centroid of the column to the load P_2 is $s = 12$ in. Also, the modulus of elasticity is $E = 29{,}000$ ksi and the yield stress is $\sigma_Y = 36$ ksi.

(a) Calculate the maximum compressive stress in the column.

(b) Determine the factor of safety with respect to yielding.



Probs. 11.6-13 and 11.6-14

Solution 11.6-13 Column with two loads

Fixed-free column. W 14 × 53

DATA

$L = 15$ ft $= 180$ in. $L_e = 2L = 360$ in.
$P_1 = 120$ k $P_2 = 40$ k $s = 12$ in.
$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

$$P = P_1 + P_2 = 160\text{ k} \quad e = \frac{P_2 s}{P} = 3.0\text{ in.}$$

$A = 15.6$ in.^{2} $I = I_1 = 541$ in.^{4} $d = 13.92$ in.

$$r^2 = \frac{I}{A} = 34.68\text{ in.}^2 \quad r = 5.889\text{ in.}$$

$$c = \frac{d}{2} = 6.960\text{ in.} \quad \frac{ec}{r^2} = 0.6021$$

$$\frac{P}{A} = 10.26\text{ ksi} \quad \frac{L_e}{2r}\sqrt{\frac{P}{EA}} = 0.5748$$

(a) MAXIMUM COMPRESSIVE STRESS

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

Substitute into Eq. (1): $\sigma_{max} = 17.6$ ksi ←

(b) FACTOR OF SAFETY WITH RESPECT TO YIELDING

$\sigma_{max} = \sigma_Y = 36$ ksi $P = P_Y$

Substitute into Eq. (1):

$$36 = \frac{P_Y}{15.6}[1 + 0.6021\sec(0.04544\sqrt{P_Y})]$$

Solve numerically: $P_Y = 302.6$ k

$$P = 160\text{ k} \quad n = \frac{P_Y}{P} = \frac{302.6\text{ k}}{160\text{ k}} = 1.89 \quad ←$$

Problem 11.6-14 A wide-flange column with a bracket is fixed at the base and free at the top (see figure on the preceding page). The column supports a load $P_1 = 75$ k acting at the centroid and a load $P_2 = 25$ k acting on the bracket at distance $s = 10.0$ in. from the load P_1. The column is a W 12 × 35 shape with $L = 16$ ft, $E = 29 \times 10^3$ ksi, and $\sigma_Y = 42$ ksi.

(a) What is the maximum compressive stress in the column?

(b) If the load P_1 remains at 75 k, what is the largest permissible value of the load P_2 in order to maintain a factor of safety of 1.8 with respect to yielding?

Solution 11.6-14 Column with two loads

Fixed-free column. W 12 × 35

DATA

$L = 16$ ft $= 192$ in. $L_e = 2L = 384$ in.
$P_1 = 75$ k $P_2 = 25$ k $s = 10.0$ in.
$E = 29{,}000$ ksi $\sigma_Y = 42$ ksi

$$P = P_1 + P_2 = 100\text{ k} \quad e = \frac{P_2 s}{P} = 2.5\text{ in.}$$

$A = 10.3$ in.^{2} $I = I_1 = 285$ in.^{4} $d = 12.50$ in.

$$r^2 = \frac{I}{A} = 27.67\text{ in.}^2 \quad r = 5.260\text{ in.}$$

$$c = \frac{d}{2} = 6.25\text{ in.} \quad \frac{ec}{r^2} = 0.5647$$

$$\frac{P}{A} = 9.709\text{ ksi} \quad \frac{L_e}{2r}\sqrt{\frac{P}{EA}} = 0.6679$$

(a) MAXIMUM COMPRESSIVE STRESS

Secant formula (Eq. 11-59):

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{EA}}\right)\right] \quad (1)$$

Substitute into Eq. (1): $\sigma_{max} = 16.7$ ksi ←

(b) LARGEST VALUE OF LOAD P_2

$P_1 = 75$ k (no change)

$m = 1.8$ with respect to yielding

Units: kips, inches

$P = P_1 + P_2 = 75 + P_2$

$$e = \frac{P_2 s}{P} = \frac{P_2(10.0)}{75 + P_2} \quad \frac{ec}{r^2} = \frac{2.259\,P_2}{75 + P_2}$$

$\sigma_{max} = \sigma_Y = 42$ ksi $P_Y = n\,P = 1.8\,(75 + P_2)$

Use Eq. (1) with σ_{max} replaced by σ_Y and P replaced by P_Y:

$$\sigma_Y = \frac{P_Y}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P_Y}{EA}}\right)\right] \quad (2)$$

Substitute into Eq. (2):

$$42 = \frac{1.8(75 + P_2)}{10.3}\left[1 + \frac{2.259\,P_2}{75 + P_2}\sec\left(0.08961\sqrt{75 + P_2}\right)\right]$$

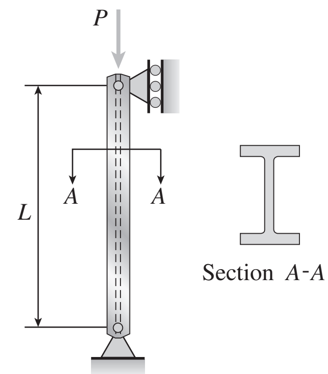
Solve numerically: $P_2 = 34.3$ k ←

Design Formulas for Columns

The problems for Section 11.9 are to be solved assuming that the axial loads are centrally applied at the ends of the columns. Unless otherwise stated, the columns may buckle in any direction.

STEEL COLUMNS

Problem 11.9-1 Determine the allowable axial load P_{allow} for a W 10 × 45 steel wide-flange column with pinned ends (see figure) for each of the following lengths: L = 8 ft, 16 ft, 24 ft, and 32 ft. (Assume E = 29,000 ksi and σ_Y = 36 ksi.)



Probs. 11.9-1 through 11.9-6

Solution 11.9-1 **Steel wide-flange column**

Pinned ends (K = 1).
Buckling about axis 2-2 (see Table E-1).
Use AISC formulas.
W 10 × 45 $A = 13.3 \text{ in.}^2$ $r_2 = 2.01$ in.
E = 29,000 ksi σ_Y = 36 ksi $\left(\frac{L}{r}\right)_{max} = 200$

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\ r = 253.5$ in. = 21.1 ft

L	8 ft	16 ft	24 ft	32 ft
L/r	47.76	95.52	143.3	191.0
n_1 (Eq. 11-79)	1.802	1.896	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.5152	0.3760	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2020	0.1137
σ_{allow} (ksi)	18.55	13.54	7.274	4.091
$P_{allow} = A\ \sigma_{allow}$	247 k	180 k	96.7 k	54.4 k

Problem 11.9-2 Determine the allowable axial load P_{allow} for a W 12 × 87 steel wide-flange column with pinned ends (see figure) for each of the following lengths: L = 10 ft, 20 ft, 30 ft, and 40 ft. (Assume E = 29,000 ksi and σ_Y = 50 ksi.)

Solution 11.9-2 **Steel wide-flange column**

Pinned ends (K = 1).
Buckling about axis 2-2 (see Table E-1).
Use AISC formulas.
W 12 × 87 $A = 25.6 \text{ in.}^2$ $r_2 = 3.07$ in.
E = 29,000 ksi σ_Y = 50 ksi $\left(\frac{L}{r}\right)_{max} = 200$

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 107.0$

$L_c = 1.070\ r = 328.5$ in. = 27.4 ft

L	10 ft	20 ft	30 ft	40 ft
L/r	39.09	78.18	117.3	156.4
n_1 (Eq. 11-79)	1.798	1.892	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.5192	0.3875	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2172	0.1222
σ_{allow} (ksi)	25.96	19.37	10.86	6.11
$P_{allow} = A\ \sigma_{allow}$	665 k	496 k	278 k	156 k

Problem 11.9-3 Determine the allowable axial load P_{allow} for a W 10 × 60 steel wide-flange column with pinned ends (see figure) for each of the following lengths: $L = 10$ ft, 20 ft, 30 ft, and 40 ft. (Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)

Solution 11.9-3 Steel wide-flange column

Pinned ends ($K = 1$).
Buckling about axis 2-2 (see Table E-1).
Use AISC formulas.
W 10 × 60 $A = 17.6 \text{ in.}^2$ $r_2 = 2.57$ in.

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi $\left(\frac{L}{r}\right)_{max} = 200$

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\, r = 324.1 \text{ in.} = 27.0 \text{ ft}$

L	10 ft	20 ft	30 ft	40 ft
L/r	46.69	93.39	140.1	186.8
n_1 (Eq. 11-79)	1.799	1.894	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.5177	0.3833	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2114	0.1189
σ_{allow} (ksi)	18.64	13.80	7.610	4.281
$P_{allow} = A\,\sigma_{allow}$	328 k	243 k	134 k	75.3 k

Problem 11.9-4 Select a steel wide-flange column of nominal depth 10 in. (W 10 shape) to support an axial load $P = 180$ k (see figure). The column has pinned ends and length $L = 14$ ft. Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi. (*Note:* The selection of columns is limited to those listed in Table E-1, Appendix E.)

Solution 11.9-4 Select a column of W10 shape

$P = 180$ k $L = 14 \text{ ft} = 168$ in. $K = 1$
$\sigma_Y = 36$ ksi
$E = 29{,}000$ ksi

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

(1) Trial value of σ_{allow}

Upper limit: use Eq. (11-81) with $L/r = 0$

$$\text{max. } \sigma_{allow} = \frac{\sigma_Y}{n_1} = \frac{\sigma_Y}{5/3} = 21.6 \text{ ksi}$$

Try $\sigma_{allow} = 16$ ksi

(2) Trial value of area

$$A = \frac{P}{\sigma_{allow}} = \frac{180\text{ k}}{16\text{ ksi}} = 11.25 \text{ in.}^2$$

(3) Trial column W 10 × 45

$A = 13.3 \text{ in.}^2$ $r = 2.01$ in.

(4) Allowable stress for trial column

$$\frac{L}{r} = \frac{168 \text{ in.}}{2.01 \text{ in.}} = 83.58 \quad \frac{L}{r} < \left(\frac{L}{r}\right)_c$$

Eqs. (11-79) and (11-81): $n_1 = 1.879$

$$\frac{\sigma_{allow}}{\sigma_Y} = 0.4153 \quad \sigma_{allow} = 14.95 \text{ ksi}$$

(5) Allowable load for trial column

$P_{allow} = \sigma_{allow} A = 199 \text{ k} > 180 \text{ k}$ (ok)
(W 10 × 45)

(6) Next smaller size column

W 10 × 30 $A = 8.84 \text{ in.}^2$ $r = 1.37$ in.

$$\frac{L}{r} = 122.6 \quad < \left(\frac{L}{r}\right)_c$$

$n = 1.916$ $\sigma_{allow} = 9.903$ ksi
$P_{allow} = 88 \text{ k} < P = 180$ k (Not satisfactory)

Problem 11.9-5 Select a steel wide-flange column of nominal depth 12 in. (W 12 shape) to support an axial load $P = 175$ k (see figure). The column has pinned ends and length $L = 35$ ft. Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi. (*Note:* The selection of columns is limited to those listed in Table E-1, Appendix E.)

Solution 11.9-5 Select a column of W12 shape

$P = 175$ k $\quad L = 35$ ft $= 420$ in. $\quad K = 1$

$\sigma_Y = 36$ ksi $\quad E = 29{,}000$ ksi

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

(1) TRIAL VALUE OF σ_{allow}

Upper limit: use Eq. (11-81) with $L/r = 0$

$$\max.\sigma_{allow} = \frac{\sigma_Y}{n_1} = \frac{\sigma_Y}{5/3} = 21.6\,\text{ksi}$$

Try $\sigma_{allow} = 8$ ksi (Because column is very long)

(2) TRIAL VALUE OF AREA

$$A = \frac{P}{\sigma_{allow}} = \frac{175\,\text{k}}{8\,\text{ksi}} = 22\,\text{in.}^2$$

(3) TRIAL COLUMN W 12 × 87

$A = 25.6$ in.^{2} $\quad r = 3.07$ in.

(4) ALLOWABLE STRESS FOR TRIAL COLUMN

$$\frac{L}{r} = \frac{4.20\,\text{in.}}{3.07\,\text{in.}} = 136.8 \qquad \frac{L}{r} > \left(\frac{L}{r}\right)_c$$

Eqs. (11-80) and (11-82): $\quad n_2 = 1.917$

$$\frac{\sigma_{allow}}{\sigma_Y} = 0.2216 \qquad \sigma_{allow} = 7.979\,\text{ksi}$$

(5) ALLOWABLE LOAD FOR TRIAL COLUMN

$P_{allow} = \sigma_{allow} A = 204$ k > 175 k $\quad$ (ok)

(6) NEXT SMALLER SIZE COLUMN

W 12 × 50 $\quad A = 14.7$ in.^{2} $\quad r = 1.96$ in.

$\frac{L}{r} = 214$ $\quad$ Since the maximum permissible value of L/r is 200, this section is not satisfactory.

Select W 12 × 87 ⟵

Problem 11.9-6 Select a steel wide-flange column of nominal depth 14 in. (W 14 shape) to support an axial load $P = 250$ k (see figure). The column has pinned ends and length $L = 20$ ft. Assume $E = 29{,}000$ ksi and $\sigma_Y = 50$ ksi. (*Note:* The selection of columns is limited to those listed in Table E-1, Appendix E.)

Solution 11.9-6 Select a column of W14 shape

$P = 250$ k $\quad L = 20$ ft $= 240$ in. $\quad K = 1$

$\sigma_Y = 50$ ksi

$E = 29{,}000$ ksi

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 107.0$

(1) TRIAL VALUE OF σ_{allow}

Upper limit: use Eq. (11-81) with $L/r = 0$

$$\max.\sigma_{allow} = \frac{\sigma_Y}{n_1} = \frac{\sigma_Y}{5/3} = 30\,\text{ksi}$$

Try $\sigma_{allow} = 12$ ksi

(2) TRIAL VALUE OF AREA

$$A = \frac{P}{\sigma_{allow}} = \frac{250\,\text{k}}{12\,\text{ksi}} = 21\,\text{in.}^2$$

(3) TRIAL COLUMN W 14 × 82

$A = 24.1$ in.^{2} $\quad r = 2.48$ in.

(4) ALLOWABLE STRESS FOR TRIAL COLUMN

$$\frac{L}{r} = \frac{240\,\text{in.}}{2.48\,\text{in.}} = 96.77 \qquad \frac{L}{r} < \left(\frac{L}{r}\right)_c$$

Eqs. (11-79) and (11-81): $\quad n_1 = 1.913$

$$\frac{\sigma_{allow}}{\sigma_Y} = 0.3089 \qquad \sigma_{allow} = 15.44\,\text{ksi}$$

(5) ALLOWABLE LOAD FOR TRIAL COLUMN

$P_{allow} = \sigma_{allow} A = 372 \text{ k} > 250 \text{ k}$ (ok)
(W 14 × 82)

(6) NEXT SMALLER SIZE COLUMN

W 14 × 53 $A = 15.6 \text{ in.}^2$ $r = 1.92$ in.

$\frac{L}{r} = 125.0 > \left(\frac{L}{r}\right)_c$
$n = 1.917$ $\sigma_{allow} = 9.557$ ksi
$P_{allow} = 149 \text{ k} < P = 250 \text{ k}$ (Not satisfactory)
Select W 14 × 82 ⟵

Problem 11.9-7 Determine the allowable axial load P_{allow} for a steel *pipe column with pinned ends* having an outside diameter of 4.5 in. and wall thickness of 0.237 in. for each of the following lengths: $L = 6$ ft, 12 ft, 18 ft, and 24 ft. (Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)

Solution 11.9-7 Steel pipe column

Pinned ends ($K = 1$).
Use AISC formulas.
$d_2 = 4.5$ in. $t = 0.237$ in. $d_1 = 4.026$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 3.1740 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 7.2326 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.5095 \text{ in.} \qquad \left(\frac{L}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

Eq.(11-76): $$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$L_c = 126.1\, r = 190.4 \text{ in.} = 15.9 \text{ ft}$

L	6 ft	12 ft	18 ft	24 ft
L/r	47.70	95.39	143.1	190.8
n_1 (Eq. 11-79)	1.802	1.896	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.5153	0.3765	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2026	0.1140
σ_{allow} (ksi)	18.55	13.55	7.293	4.102
$P_{allow} = A\,\sigma_{allow}$	58.9 k	43.0 k	23.1 k	13.0 k

Problem 11.9-8 Determine the allowable axial load P_{allow} for a steel *pipe column with pinned ends* having an outside diameter of 220 mm and wall thickness of 12 mm for each of the following lengths: $L = 2.5$ m, 5 m, 7.5 m, and 10 m. (Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.)

Solution 11.9-8 Steel pipe column

Pinned ends ($K = 1$).
Use AISC formulas.
$d_2 = 220$ mm $t = 12$ mm $d_1 = 196$ mm

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 7841.4 \text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 42.548 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 73.661 \text{ mm} \qquad \left(\frac{L}{r}\right)_{max} = 200$$

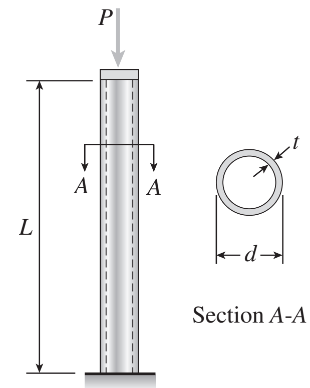
$E = 200$ GPa $\sigma_Y = 250$ MPa

Eq.(11-76): $$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$$

$L_c = 125.7\, r = 9257 \text{ mm} = 9.26 \text{ m}$

L	2.5 m	5.0 m	7.5 m	10.0 m
L/r	33.94	67.88	101.8	135.8
n_1 (Eq. 11-79)	1.765	1.850	1.904	–
n_2 (Eq. 11-80)	–	–	–	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.5458	0.4618	0.3528	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–	0.2235
σ_{allow} (MPa)	136.4	115.5	88.20	55.89
$P_{allow} = A\,\sigma_{allow}$	1070 kN	905 kN	692 kN	438 kN

Problem 11.9-9 Determine the allowable axial load P_{allow} for a steel pipe column that is fixed at the base and free at the top (see figure) for each of the following lengths: $L = 6$ ft, 9 ft, 12 ft, and 15 ft. The column has outside diameter $d = 6.625$ in. and wall thickness $t = 0.280$ in. (Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-9 through 11.9-12

Solution 11.9-9 **Steel pipe column**

Fixed-free column ($K = 2$).

Use AISC formulas.

$d_2 = 6.625$ in. $\quad t = 0.280$ in. $\quad d_1 = 6.065$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 5.5814\ \text{in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 28.142\ \text{in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2455 \qquad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\quad \sigma_Y = 36$ ksi

Eq.(11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\,\frac{r}{k} = 141.6\ \text{in.} = 11.8\ \text{ft}$

L	6 ft	9 ft	12 ft	15 ft
KL/r	64.13	96.19	128.3	160.3
n_1 (Eq. 11-79)	1.841	1.897	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.4730	0.3737	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2519	0.1614
σ_{allow} (ksi)	17.03	13.45	9.078	5.810
$P_{allow} = A\,\sigma_{allow}$	95.0 k	75.1 k	50.7 k	32.4 k

Problem 11.9-10 Determine the allowable axial load P_{allow} for a steel pipe column that is fixed at the base and free at the top (see figure) for each of the following lengths: $L = 2.6$ m, 2.8 m, 3.0 m, and 3.2 m. The column has outside diameter $d = 140$ mm and wall thickness $t = 7$ mm. (Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.)

Solution 11.9-10 **Steel pipe column**

Fixed-free column ($K = 2$).

Use AISC formulas.

$d_2 = 140$ mm $\quad t = 7.0$ mm $\quad d_1 = 126$ mm

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2924.8\ \text{mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 6.4851 \times 10^6\ \text{mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.09\ \text{mm} \qquad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 200$ GPa $\quad \sigma_Y = 250$ MPa

Eq.(11-76): $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$

$L_c = 125.7\,\frac{r}{K} = 2959\ \text{mm} = 2.959\ \text{m}$

L	2.6 m	2.8 m	3.0 m	3.2 m
KL/r	110.4	118.9	127.4	135.9
n_1 (Eq. 11-79)	1.911	1.916	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	0.3212	0.2882	–	–
σ_{allow}/σ_Y (Eq. 11-82)	–	–	0.2537	0.2230
σ_{allow} (MPa)	80.29	72.06	63.43	55.75
$P_{allow} = A\,\sigma_{allow}$	235 kN	211 kN	186 kN	163 kN

Problem 11.9-11 Determine the maximum permissible length L_{max} for a steel pipe column that is fixed at the base and free at the top and must support an axial load $P = 40$ k (see figure). The column has outside diameter $d = 4.0$ in., wall thickness $t = 0.226$ in., $E = 29{,}000$ ksi, and $\sigma_Y = 42$ ksi.

Solution 11.9-11 Steel pipe column

Fixed-free column ($K = 2$). $P = 40$ k

Use AISC formulas.

$d_2 = 4.0$ in. $t = 0.226$ in. $d_1 = 3.548$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.6795\text{ in.}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 4.7877\text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.3367 \qquad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\sigma_Y = 42$ ksi

$$\text{Eq.(11-76):} \quad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$$

$$L_c = 116.7\,\frac{r}{K} = 78.03\text{ in.} = 6.502\text{ ft}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).
If $L > L_c$, use Eqs. (11-80) and (11-82).

L(ft)	5.20	5.25	5.23
KL/r	93.86	94.26	93.90
n_1 (Eq. 11-79)	1.903	1.904	1.903
n_2 (Eq. 11-80)	–	–	–
σ_{allow}/σ_Y (Eq. 11-81)	0.3575	0.3541	0.3555
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–
σ_{allow} (ksi)	15.02	14.87	14.93
$P_{allow} = A\,\sigma_{allow}$	40.2 k	39.8 k	40.0 k

For $P = 40$ k, $L_{max} = 5.23$ ft ←

Problem 11.9-12 Determine the maximum permissible length L_{max} for a steel pipe column that is fixed at the base and free at the top and must support an axial load $P = 500$ kN (see figure). The column has outside diameter $d = 200$ mm, wall thickness $t = 10$ mm, $E = 200$ GPa, and $\sigma_Y = 250$ MPa.

Solution 11.9-12 Steel pipe column

Fixed-free column ($K = 2$). $P = 500$ kN

Use AISC formulas.

$d_2 = 200$ mm $t = 10$ mm $d_1 = 180$ mm

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 5{,}969.0\text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 27.010 \times 10^6\text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 67.27\text{ mm} \quad \left(\frac{KL}{r}\right)_{max} = 200$$

$E = 200$ GPa $\sigma_Y = 250$ MPa

$$\text{Eq. (11-76):} \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$$

$$L_c = 125.7\,\frac{r}{K} = 4.226\text{ m}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).
If $L > L_c$, use Eqs. (11-80) and (11-82).

L(m)	3.55	3.60	3.59
KL/r	105.5	107.0	106.7
n_1 (Eq. 11-79)	1.908	1.909	1.909
n_2 (Eq. 11-80)	–	–	–
σ_{allow}/σ_Y (Eq. 11-81)	0.3393	0.3338	0.3349
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–
σ_{allow} (MPa)	84.83	83.46	83.74
$P_{allow} = A\,\sigma_{allow}$	506 kN	498 kN	500 kN

For $P = 500$ kN, $L = 3.59$ m ←

Problem 11.9-13 A steel pipe column with *pinned ends* supports an axial load $P = 21$ k. The pipe has outside and inside diameters of 3.5 in. and 2.9 in., respectively. What is the maximum permissible length L_{max} of the column if $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi?

Solution 11.9-13 Steel pipe column

Pinned ends ($K = 1$). $P = 21$ k

Use AISC formulas.

$d_2 = 3.5$ in. $t = 0.3$ in. $d_1 = 2.9$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 3.0159 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.8943 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.1363 \text{ in.} \qquad \left(\frac{L}{r}\right)_{max} = 200$$

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$L_c = 126.1\, r = 143.3$ in. $= 11.9$ ft

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

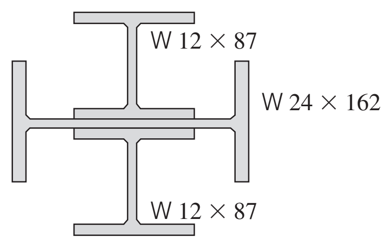
If $L > L_c$, use Eqs. (11-80) and (11-82).

L(ft)	13.8	13.9	14.0
L/r	145.7	146.8	147.8
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	–	–	–
σ_{allow}/σ_Y (Eq. 11-82)	0.1953	0.1925	0.1898
σ_{allow} (ksi)	7.031	6.931	6.832
$P_{allow} = A\,\sigma_{allow}$	21.2 k	20.9 k	20.6 k

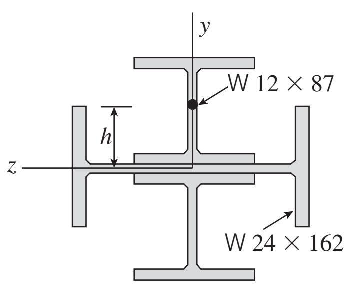
For $P = 21$ k, $L = 13.9$ ft ←

Problem 11.9-14 The steel columns used in a college recreation center are 55 ft long and are formed by welding three wide-flange sections (see figure). The columns are pin-supported at the ends and may buckle in any direction.

Calculate the allowable load P_{allow} for one column, assuming $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.



Solution 11.9-14 Pinned-end column ($K = 1$)



$L = 55$ ft $= 660$ in.

$E = 29{,}000$ ksi

$\sigma_Y = 36$ ksi

W 12 × 87

$A = 25.6$ in.^{2} $d = 12.53$ in.

$I_1 = 740$ in.^{4} $I_2 = 241$ in.^{4}

W 24 × 162

$A = 47.7$ in.^{2} $t_w = 0.705$ in.

$I_1 = 5170$ in.^{4} $I_2 = 443$ in.^{4}

For the entire cross section

$A = 2\,(25.6) + 47.7 = 98.9$ in.^{2}

$I_Y = 2\,(241) + 5170 = 5652$ in.^{4}

$h = d/2 + t_w/2 = 6.6175$ in.

$I_z = 443 + 2\,[740 + (25.6)(6.6175)^2] = 4165$ in.^{4}

$$\text{min. } r = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{4165}{98.9}} = 6.489 \text{ in.}$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$$\frac{L}{r} = \frac{660 \text{ in.}}{6.489 \text{ in.}} = 101.7 \qquad \frac{L}{r} < \left(\frac{L}{r}\right)_c$$

$\therefore$ Use Eqs. (11-79) and (11-81).

From Eq. (11-79): $n_1 = 1.904$

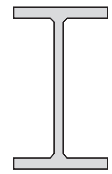
From Eq. (11-81): $\sigma_{allow}/\sigma_Y = 0.3544$

$\sigma_{allow} = 0.3544\,\sigma_Y = 12.76$ ksi

$P_{allow} = \sigma_{allow} A = (12.76 \text{ ksi})(98.9 \text{ in.}^2)$

$= 1260$ k ←

Problem 11.9-15 A W 8 × 28 steel wide-flange column with pinned ends carries an axial load P. What is the maximum permissible length L_{max} of the column if (a) $P = 50$ k, and (b) $P = 100$ k? (Assume $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-15 and 11.9-16

Solution 11.9-15 Steel wide-flange column

Pinned ends ($K = 1$).

Buckling about axis 2-2 (see Table E-1).

Use AISC formulas.

W 8 × 28 $\quad A = 8.25$ in.^{2} $\quad r_2 = 1.62$ in.

$E = 29{,}000$ ksi $\quad \sigma_Y = 36$ ksi $\quad \left(\frac{L}{r}\right)_{max} = 200$

Eq. (11-76): $\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$

$L_c = 126.1\,r = 204.3$ in. $= 17.0$ ft

For each load P, select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

L (ft)	21.0	21.5	21.2
L/r	155.6	159.3	157.0
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	–	–	–
σ_{allow}/σ_Y (Eq. 11-82)	0.1714	0.1635	0.1682
σ_{allow} (ksi)	6.171	5.888	6.056
$P_{allow} = A\,\sigma_{allow}$	50.9 k	48.6 k	50.0 k

(a) $P = 50$ k

For $P = 50$ k, $\quad L_{max} = 21.2$ ft ←

(b) $P = 100$ k

L (ft)	14.3	14.4	14.5	
L/r	105.9	106.7	107.4	
n_1 (Eq. 11-79)	1.908	1.908	1.909	
n_2 (Eq. 11-80)	–	–	–	
σ_{allow}/σ_Y (Eq. 11-81)	0.3393	0.3366	0.3338	
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–	
σ_{allow} (ksi)	12.21	12.12	12.02	
$P_{allow} = A\,\sigma_{allow}$	100.8 k	100.0 k	99.2 k	k

For $P = 100$ k, $\quad L_{max} = 14.4$ ft ←

Problem 11.9-16 A W 10 × 45 steel wide-flange column with pinned ends carries an axial load P. What is the maximum permissible length L_{max} of the column if (a) $P = 125$ k, and (b) $P = 200$ k? (Assume $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi.)

Solution 11.9-16 **Steel wide-flange column**

Pinned ends ($K = 1$).
Buckling about axis 2-2 (see Table E-1).
Use AISC formulas.
W 10 × 45 $A = 13.3\text{ in.}^2$ $r_2 = 2.01$ in.

$E = 29{,}000$ ksi $\sigma_Y = 42$ ksi $\left(\dfrac{L}{r}\right)_{max} = 200$

Eq. (11-76): $$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$$

$L_c = 116.7\, r = 235$ in. $= 19.6$ ft

For each load P, select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{allow} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).
If $L > L_c$, use Eqs. (11-80) and (11-82).

(a) $P = 125$ k

L (ft)	21.0	21.1	21.2
L/r	125.4	126.0	126.6
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
σ_{allow}/σ_Y (Eq. 11-81)	–	–	–
σ_{allow}/σ_Y (Eq. 11-82)	0.2202	0.2241	0.2220
σ_{allow} (ksi)	9.500	9.411	9.322
$P_{allow} = A\,\sigma_{allow}$	126.4 k	125.2 k	124.0 k

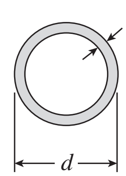
For $P = 125$ k, $L_{max} = 21.1$ ft ←

(b) $P = 200$ k

L(ft)	15.5	15.6	15.7
L/r	92.54	93.13	93.73
n_1 (Eq. 11-79)	1.902	1.902	1.903
n_2 (Eq. 11-80)	–	–	–
σ_{allow}/σ_Y (Eq. 11-81)	0.3607	0.3584	0.3561
σ_{allow}/σ_Y (Eq. 11-82)	–	–	–
σ_{allow} (ksi)	15.15	15.05	14.96
$P_{allow} = A\,\sigma_{allow}$	201.5 k	200.2 k	198.9 k

For $P = 200$ k, $L_{max} = 15.6$ ft ←

Problem 11.9-17 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 20$ ft that is pinned at both ends and must support an axial load $P = 25$ k. Assume that the wall thickness t is equal to $d/20$. (Use $E = 29{,}000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-17 through 11.9-20

Solution 11.9-17 **Pipe column**

Pinned ends ($K = 1$).

$L = 20$ ft $= 240$ in. $P = 25$ k

d = outside diameter $t = d/20$

$E = 29{,}000$ ksi $\sigma_Y = 36$ ksi

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] = 0.14923\, d^2$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] = 0.016881\, d^4$$

$$r = \sqrt{\frac{I}{A}} = 0.33634\, d$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1 \qquad L_c = (126.1)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

For $P = 25$ k, $d = 4.89$ in. ←

d (in.)	4.80	4.90	5.00
A (in.^{2})	3.438	3.583	3.731
I (in.^{4})	8.961	9.732	10.551
r (in.)	1.614	1.648	1.682
L_c (in.)	204	208	212
L/r	148.7	145.6	142.7
n_2 (Eq. 11-80)	23/12	23/12	23/12
σ_{allow}/σ_Y (Eq. 11-82)	0.1876	0.1957	0.2037
σ_{allow} (ksi)	6.754	7.044	7.333
$P_{allow} = A\,\sigma_{allow}$	23.2 k	25.2 k	27.4 k

Problem 11.9-18 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 3.5$ m that is pinned at both ends and must support an axial load $P = 130$ kN. Assume that the wall thickness t is equal to $d/20$. (Use $E = 200$ GPa and $\sigma_Y = 275$ MPa).

Solution 11.9-18 Pipe column

Pinned ends ($K = 1$).

$L = 3.5$ m $\quad P = 130$ kN

d = outside diameter $\quad t = d/20$

$E = 200$ GPa $\quad \sigma_Y = 275$ MPa

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] = 0.14923\, d^2$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] = 0.016881\, d^4$$

$$r = \sqrt{\frac{I}{A}} = 0.33634\, d$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 119.8 \qquad L_c = (119.8)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

d (mm)	98	99	100
A (mm^2)	1433	1463	1492
I (mm^4)	1557×10^3	1622×10^3	1688×10^3
r (mm)	32.96	33.30	33.64
L_c (mm)	3950	3989	4030
L/r	106.2	105.1	104.0
n_1 (Eq. 11-79)	1.912	1.911	1.910
σ_{allow}/σ_Y (Eq. 11-81)	0.3175	0.3219	0.3263
σ_{allow} (MPa)	87.32	88.53	89.73
$P_{allow} = A\,\sigma_{allow}$	125.1 kN	129.5 kN	133.9 kN

For $P = 130$ kN, $d = 99$ mm ←

Problem 11.9-19 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 11.5$ ft that is pinned at both ends and must support an axial load $P = 80$ k. Assume that the wall thickness t is 0.30 in. (Use $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi.)

Solution 11.9-19 Pipe column

Pinned ends ($K = 1$).

$L = 11.5$ ft $= 138$ in. $\quad P = 80$ k

d = outside diameter $\quad t = 0.30$ in.

$E = 29{,}000$ ksi $\quad \sigma_Y = 42$ ksi

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2]$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] \qquad r = \sqrt{\frac{I}{A}}$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7 \qquad L_c = (116.7)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \le L_c$, Use Eqs. (11-79) and (11-81).
If $L \ge L_c$, Use Eqs. (11-80) and (11-82).

For $P = 80$ k, $d = 5.23$ in. ←

d (in.)	5.20	5.25	5.30
A (in.^{2})	4.618	4.665	4.712
I (in.^{4})	13.91	14.34	14.78
r (in.)	1.736	1.753	1.771
L_c (in.)	203	205	207
L/r	79.49	78.72	77.92
n_1 (Eq. 11-79)	1.883	1.881	1.880
σ_{allow}/σ_Y (Eq. 11-81)	0.4079	0.4107	0.4133
σ_{allow} (ksi)	17.13	17.25	17.36
$P_{allow} = A\,\sigma_{allow}$	79.1 k	80.5 k	81.8 k

Problem 11.9-20 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 3.0$ m that is pinned at both ends and must support an axial load $P = 800$ kN. Assume that the wall thickness t is 9 mm. (Use $E = 200$ GPa and $\sigma_Y = 300$ MPa.)

Solution 11.9-20 Pipe column

Pinned ends ($K = 1$).
$L = 3.0$ m $P = 800$ kN
d = outside diameter $t = 9.0$ mm
$E = 200$ GPa $\sigma_Y = 300$ MPa

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2]$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] \qquad r = \sqrt{\frac{I}{A}}$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 114.7 \qquad L_c = (114.7)r$$

Select various values of diameter d until we obtain $P_{allow} = P$.

If $L \le L_c$, Use Eqs. (11-79) and (11-81).
If $L \ge L_c$, Use Eqs. (11-80) and (11-82).

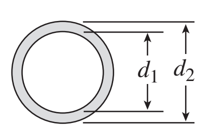
d (mm)	193	194	195
A (mm^2)	5202	5231	5259
I (mm^4)	20.08×10^6	22.43×10^6	22.80×10^6
r (mm)	65.13	65.48	65.84
L_c (mm)	7470	7510	7550
L/r	46.06	45.82	45.57
n_1 (Eq. 11-79)	1.809	1.809	1.808
σ_{allow}/σ_Y (Eq. 11-81)	0.5082	0.5087	0.5094
σ_{allow} (MPa)	152.5	152.6	152.8
$P_{allow} = A\,\sigma_{allow}$	793.1 kN	798.3 kN	803.8 kN

For $P = 800$ kn, $d = 194$ mm ←

Aluminum Columns

Problem 11.9-21 An aluminum pipe column (alloy 2014-T6) with pinned ends has outside diameter $d_2 = 5.60$ in. and inside diameter $d_1 = 4.80$ in. (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 6$ ft, 8 ft, 10 ft, and 12 ft.



Probs. 11.9-21 through 11.9-24

Solution 11.9-21 Aluminum pipe column

Alloy 2014-T6
Pinned ends ($K = 1$).
$d_2 = 5.60$ in.
$d_1 = 4.80$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 6.535 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 22.22 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.844 \text{ in.}$$

Use Eqs. (11-84 *a* and *b*):

$\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi $\quad L/r \leq 55$

$\sigma_{allow} = 54{,}000/(L/r)^2$ ksi $\quad L/r \geq 55$

L (ft)	6 ft	8 ft	10 ft	12 ft
L/r	39.05	52.06	65.08	78.09
σ_{allow} (ksi)	21.72	18.73	12.75	8.86
$P_{allow} = \sigma_{allow} A$	142 k	122 k	83 k	58 k

Problem 11.9-22 An aluminum pipe column (alloy 2014-T6) with pinned ends has outside diameter $d_2 = 120$ mm and inside diameter $d_1 = 110$ mm (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 1.0$ m, 2.0 m, 3.0 m, and 4.0 m.

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-22 Aluminum pipe column

Alloy 2014-T6

Pinned ends ($K = 1$).

$d_2 = 120$ mm $= 4.7244$ in.

$d_1 = 110$ mm $= 4.3307$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.800 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 7.188 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 40.697 \text{ mm} = 1.6022 \text{ in.}$$

Use Eqs. (11-84 *a* and *b*):

$\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi $\quad L/r \leq 55$

$\sigma_{allow} = 54{,}000/(L/r)^2$ ksi $\quad L/r \geq 55$

L (m)	1.0 m	2.0 m	3.0 m	4.0 m
L (in.)	39.37	78.74	118.1	157.5
L/r	24.58	49.15	73.73	98.30
σ_{allow} (ksi)	25.05	19.40	9.934	5.588
$P_{allow} = \sigma_{allow} A$	70.14 k	54.31 k	27.81 k	15.65 k
P_{allow} (kN)	312 kN	242 kN	124 kN	70 kN

Problem 11.9-23 An aluminum pipe column (alloy 6061-T6) that is fixed at the base and free at the top has outside diameter $d_2 = 3.25$ in. and inside diameter $d_1 = 3.00$ in. (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 2$ ft, 3 ft, 4 ft, and 5 ft.

Solution 11.9-23 Aluminum pipe column

Alloy 6061-T6

Fixed-free ends ($K = 2$).

$d_2 = 3.25$ in.

$d_1 = 3.00$ in.

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.227 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 1.500 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.106 \text{ in.}$$

Use Eqs. (11-85 *a* and *b*):

$\sigma_{allow} = 20.2 - 0.126\,(KL/r)$ ksi $\quad KL/r \leq 66$

$\sigma_{allow} = 51{,}000/(KL/r)^2$ ksi $\quad KL/r \geq 66$

L (ft)	2 ft	3 ft	4 ft	5 ft
KL/r	43.40	65.10	86.80	108.5
σ_{allow} (ksi)	14.73	12.00	6.77	4.33
$P_{allow} = \sigma_{allow} A$	18.1 k	14.7 k	8.3 k	5.3 k

Problem 11.9-24 An aluminum pipe column (alloy 6061-T6) that is fixed at the base and free at the top has outside diameter $d_2 = 80$ mm and inside diameter $d_1 = 72$ mm (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 0.6$ m, 0.8 m, 1.0 m, and 1.2 m.

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-24 Aluminum pipe column

Alloy 6061-T6

Fixed-free ends ($K = 2$).

$d_2 = 80 \text{ mm} = 3.1496 \text{ in.}$

$d_1 = 72 \text{ mm} = 2.8346 \text{ in.}$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.480 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^2 - d_1^2) = 1.661 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 26.907 \text{ mm} = 1.059 \text{ in.}$$

Use Eqs. (11-85 *a* and *b*):

$\sigma_{allow} = 20.2 - 0.126\,(KL/r)$ ksi $\quad KL/r \leq 66$

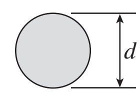
$\sigma_{allow} = 51{,}000/(KL/r)^2$ ksi $\quad KL/r \geq 66$

L (m)	0.6 m	0.8 m	1.0 m	1.2 m
KL (in.)	47.24	62.99	78.74	94.49
KL/r	44.61	59.48	74.35	89.23
σ_{allow} (ksi)	14.58	12.71	9.226	6.405
$P_{allow} = \sigma_{allow} A$	21.58 k	18.81 k	13.65 k	9.48 k
P_{allow} (kN)	96 kN	84 kN	61 kN	42 kN

Problem 11.9-25 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 60$ k. The bar has pinned supports and is made of alloy 2014-T6.

(a) If the diameter $d = 2.0$ in., what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 30$ in., what is the minimum required diameter d_{min}?



Probs. 11.9-25 through 11.9-28

Solution 11.9-25 Aluminum bar

Alloy 2014-T6

Pinned supports ($K = 1$). $\quad P = 60$ k

(a) Find L_{max} if $d = 2.0$ in.

$$A = \frac{\pi d^2}{4} = 3.142 \text{ in.}^2 \qquad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.5 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{60 \text{ k}}{3.142 \text{ in.}^2} = 19.10 \text{ ksi}$$

Assume L/r is less than 55:

Eq. (11-84*a*): $\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi

or $19.10 = 30.7 - 0.23\,(L/r)$

Solve for L/r: $\dfrac{L}{r} = 50.43 \quad \dfrac{L}{r} < 55 \quad \therefore$ ok

$L_{max} = (50.43) \quad r = 25.2$ in. ←

(b) Find d_{min} if $L = 30$ in.

$$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{30 \text{ in.}}{d/4} = \frac{120 \text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{60 \text{ k}}{\pi d^2/4} = \frac{76.39}{d^2} \text{ (ksi)}$$

Assume L/r is greater than 55:

Eq. (11-84*b*): $\sigma_{allow} = \dfrac{54{,}000 \text{ ksi}}{(L/r)^2}$

or $\quad \dfrac{76.39}{d^2} = \dfrac{54{,}000}{(120/d)^2}$

$d^4 = 20.37 \text{ in.}^4 \qquad d_{min} = 2.12$ in. ←

$L/r = 120/d = 120/2.12 = 56.6 > 55 \quad \therefore$ ok

Problem 11.9-26 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 175$ kN. The bar has pinned supports and is made of alloy 2014-T6.

(a) If the diameter $d = 40$ mm, what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 0.6$ m, what is the minimum required diameter d_{min}?

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-26 Aluminum bar

Alloy 2014-T6

Pinned supports ($K = 1$). $P = 175$ kN $= 39.34$ k

(a) FIND L_{max} IF $d = 40$ MM $= 1.575$ IN.

$$A = \frac{\pi d^2}{4} = 1.948 \text{ in.}^2 \qquad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.3938 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{39.34 \text{ k}}{1.948 \text{ in.}^2} = 20.20 \text{ ksi}$$

Assume L/r is less than 55:

Eq. (11-84*a*): $\sigma_{allow} = 30.7 - 0.23\,(L/r)$ ksi

or $20.20 = 30.7 - 0.23\,(L/r)$

Solve for L/r: $\frac{L}{r} = 45.65 \quad \frac{L}{r} < 55 \quad \therefore$ ok

$L_{max} = (45.65)\,r = 17.98$ in. $= 457$ mm ←

(b) FIND d_{min} IF $L = 0.6$ m $= 23.62$ in.

$$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{23.62 \text{ in.}}{d/4} = \frac{94.48 \text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{39.34 \text{ k}}{\pi d^2/4} = \frac{50.09}{d^2} \quad \text{(ksi)}$$

Assume L/r is greater than 55:

Eq. (11-84*b*): $\sigma_{allow} = \frac{54{,}000 \text{ ksi}}{(L/r)^2}$

or $\frac{50.09}{d^2} = \frac{54{,}000}{(94.48/d)^2}$

$d^4 = 8.280 \text{ in.}^4 \quad d_{min} = 1.696$ in. $= 43.1$ mm ←

$L/r = 94.48/d = 94.48/1.696 = 55.7 > 55 \quad \therefore$ ok

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Problem 11.9-27 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 10$ k. The bar has pinned supports and is made of alloy 6061-T6.

(a) If the diameter $d = 1.0$ in., what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 20$ in., what is the minimum required diameter d_{min}?

Solution 11.9-27 Aluminum bar

Alloy 6061-T6

Pinned Supports ($K = 1$). $P = 10$ k

(a) FIND L_{max} IF $d = 1.0$ IN.

$$A = \frac{\pi d^2}{4} = 0.7854 \text{ in.}^2 \qquad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2500 \text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{10 \text{ k}}{0.7854 \text{ in.}^2} = 12.73 \text{ ksi}$$

Assume L/r is less than 66:

Eq. (11-85*a*): $\sigma_{allow} = 20.2 - 0.126\,(L/r)$ ksi

or $12.73 = 20.2 - 0.126\,(L/r)$

Solve For L/r: $\frac{L}{r} = 59.29 \quad \frac{L}{r} < 66 \quad \therefore$ ok

$L_{max} = (59.29)r = 14.8$ in. ←

(b) FIND d_{min} IF $L = 20$ in.

$$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{20\text{ in.}}{d/4} = \frac{80\text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{10\text{ k}}{\pi d^2/4} = \frac{12.73}{d^2}\text{ (ksi)}$$

Assume L/r is Greater than 66:

Eq. (11-85b): $\sigma_{allow} = \dfrac{51{,}000\text{ ksi}}{(L/r)^2}$

or $\dfrac{12.73}{d^2} = \dfrac{51{,}000}{(80/d)^2}$

$d^4 = 1.597\text{ in.}^4 \qquad d_{min} = 1.12\text{ in.}$ ←

$L/r = 80/d = 80/1.12 = 71 > 66 \quad \therefore$ ok

Problem 11.9-28 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 60$ kN. The bar has pinned supports and is made of alloy 6061-T6.

(a) If the diameter $d = 30$ mm, what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 0.6$ m, what is the minimum required diameter d_{min}?

(*Hint:* Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-28 Aluminum bar

Alloy 6061-T6

Pinned Supports ($K = 1$). $\quad P = 60\text{ kN} = 13.49\text{ k}$

(a) FIND L_{max} IF $d = 30$ MM $= 1.181$ IN.

$$A = \frac{\pi d^2}{4} = 1.095\text{ in.}^2 \qquad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2953\text{ in.}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{13.49\text{ k}}{1.095\text{ in.}^2} = 12.32\text{ ksi}$$

Assume L/r is less than 66:

Eq. (11-85a): $\sigma_{allow} = 20.2 - 0.126\,(L/r)$ ksi

or $12.32 = 20.2 - 0.126\,(L/r)$

Solve For L/r: $\dfrac{L}{r} = 62.54 \qquad \dfrac{L}{r} < 66 \quad \therefore$ ok

$L_{max} = (62.54)r = 18.47\text{ in.} = 469\text{ mm}$ ←

(b) FIND d_{min} IF $L = 0.6$ M $= 23.62$ IN.

$$A = \frac{\pi d^2}{4} \qquad r = \frac{d}{4} \qquad \frac{L}{r} = \frac{23.62\text{ in.}}{d/4} = \frac{94.48\text{ in.}}{d}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{13.48\text{ k}}{\pi d^2/4} = \frac{17.18}{d^2}\text{ (ksi)}$$

Assume L/r is Greater than 66:

Eq. (11-85b): $\sigma_{allow} = \dfrac{51{,}000\text{ ksi}}{(L/r)^2}$

or $\dfrac{17.18}{d^2} = \dfrac{51{,}000}{(94.48/d)^2}$

$d^4 = 3.007\text{ in.}^4 \quad d_{min} = 1.317\text{ in.} = 33.4\text{ mm}$ ←

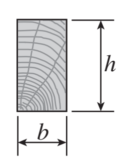
$L/r = 94.48/d = 94.48/1.317 = 72 > 66 \quad \therefore$ ok

Wood Columns

When solving the problems for wood columns, assume that the columns are constructed of sawn lumber ($c = 0.8$ and $K_{cE} = 0.3$) and have pinned-end conditions. Also, buckling may occur about either principal axis of the cross section.

Problem 11.9-29 A wood post of rectangular cross section (see figure) is constructed of 4 in. × 6 in. structural grade, Douglas fir lumber ($F_c = 2{,}000$ psi, $E = 1{,}800{,}00$ psi). The net cross-sectional dimensions of the post are $b = 3.5$ in. and $h = 5.5$ in. (see Appendix F).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 5.0$ ft, 7.5 ft, and 10.0 ft.



Probs. 11.9-29 through 11.9-32

Solution 11.9-29 **Wood post (rectangular cross section)**

$F_c = 2{,}000$ psi $E = 1{,}800{,}000$ psi $c = 0.8$

$K_{cE} = 0.3$ $b = 3.5$ in. $h = 5.5$ in. $d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

Eq. (11-92): $P_{allow} = F_cC_PA = F_cC_Pbh$

L_e	5 ft	7.5 ft	10.0 ft
L_e/d	17.14	25.71	34.29
ϕ	0.9188	0.4083	0.2297
C_P	0.6610	0.3661	0.2176
P_{allow}	25.4 k	14.1 k	8.4 k

Problem 11.9-30 A wood post of rectangular cross section (see figure) is constructed of structural grade, southern pine lumber ($F_c = 14$ MPa, $E = 12$ GPa). The cross-sectional dimensions of the post (actual dimensions) are $b = 100$ mm and $h = 150$ mm.

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 1.5$ m, 2.0 m, and 2.5 m.

Solution 11.9-30 **Wood post (rectangular cross section)**

$F_c = 14$ MPa $E = 12$ GPa $c = 0.8$ $K_{cE} = 0.3$

$b = 100$ mm $h = 150$ mm $d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

Eq. (11-92): $P_{allow} = F_cC_PA = F_cC_Pbh$

L_e	1.5 m	2.0 m	2.5 m
L_e/d	15	20	25
ϕ	1.1429	0.6429	0.4114
C_P	0.7350	0.5261	0.3684
P_{allow}	154 kN	110 kN	77 kN ←

Problem 11.9-31 A wood column of rectangular cross section (see figure) is constructed of 4 in. × 8 in. construction grade, western hemlock lumber ($F_c = 1{,}000$ psi, $E = 1{,}300{,}000$ psi). The net cross-sectional dimensions of the column are $b = 3.5$ in. and $h = 7.25$ in. (see Appendix F).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 6$ ft, 8 ft, and 10 ft.

Solution 11.9-31 **Wood column (rectangular cross section)**

$F_c = 1{,}000$ psi $E = 1{,}300{,}000$ psi $c = 0.8$

$K_{cE} = 0.3$ $b = 3.5$ in. $h = 7.25$ in. $d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

Eq. (11-92): $P_{allow} = F_cC_FA = F_cC_Pbh$

L_e	6 ft	8 ft	10 ft
L_e/d	20.57	27.43	34.29
ϕ	0.9216	0.5184	0.3318
C_P	0.6621	0.4464	0.3050
P_{allow}	16.8 k	11.3 k	7.7 k ←

Problem 11.9-32 A wood column of rectangular cross section (see figure) is constructed of structural grade, Douglas fir lumber (F_c = 12 MPa, E = 10 GPa). The cross-sectional dimensions of the column (actual dimensions) are b = 140 mm and h = 210 mm.

Determine the allowable axial load P_{allow} for each of the following lengths: L = 2.5 m, 3.5 m, and 4.5 m.

Solution 11.9-32 **Wood column (rectangular cross section)**

$F_c = 12$ MPa $\quad E = 10$ GPa $\quad c = 0.8 \quad K_{cE} = 0.3$

$b = 140$ mm $\quad h = 210$ mm $\quad d = b$

Find P_{allow}

Eq. (11-94): $\phi = \dfrac{K_{cE}E}{F_c(L_e/d)^2}$

Eq. (11-95): $C_P = \dfrac{1+\phi}{2c} - \sqrt{\left[\dfrac{1+\phi}{2c}\right]^2 - \dfrac{\phi}{c}}$

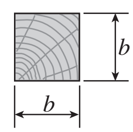
Eq. (11-92): $P_{allow} = F_cC_PA = F_cC_Pbh$

L_e	2.5 m	3.5 m	4.5 m
L_e/d	17.86	25.00	32.14
ϕ	0.7840	0.4000	0.2420
C_P	0.6019	0.3596	0.2284
P_{allow}	212 kN	127 kN	81 kN ←

Problem 11.9-33 A square wood column with side dimensions b (see figure) is constructed of a structural grade of Douglas fir for which F_c = 1,700 psi and E = 1,400,000 psi. An axial force P = 40 k acts on the column.

(a) If the dimension b = 5.5 in., what is the maximum allowable length L_{max} of the column?

(b) If the length L = 11 ft, what is the minimum required dimension b_{min}?



Probs. 11.9-33 through 11.9-36

Solution 11.9-33 **Wood column (square cross section)**

$F_c = 1{,}700$ psi $\quad E = 1{,}400{,}000$ psi $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 40$ k

(a) Maximum length L_{max} for $b = d = 5.5$ in.

From Eq. (11-92): $C_P = \dfrac{P}{F_cb^2} = 0.77783$

From Eq. (11-95):

$C_P = 0.77783 = \dfrac{1+\phi}{1.6} - \sqrt{\left[\dfrac{1+\phi}{1.6}\right]^2 - \dfrac{\phi}{0.8}}$

Trial and error: $\phi = 1.3225$

From Eq. (11-94): $\dfrac{L}{d} = \sqrt{\dfrac{K_{cE}E}{\phi F_c}} = 13.67$

$\therefore L_{max} = 13.67\ d = (13.67)(5.5\text{ in.})$

$= 75.2$ in. ←

(b) Minimum dimension b_{min} for $L = 11$ ft

Trial and error: $\dfrac{L}{d} = \dfrac{L}{b} \quad \phi = \dfrac{K_{cE}E}{F_c(L/d)^2}$

$C_P = \dfrac{1+\phi}{1.6} - \sqrt{\left[\dfrac{1+\phi}{1.6}\right]^2 - \dfrac{\phi}{0.8}} \qquad P = F_cC_Pb^2$

Given load: $P = 40$ k

Trial b (in.)	$\dfrac{L}{d} = \dfrac{L}{b}$	ϕ	C_P	P (kips)
6.50	20.308	0.59907	0.49942	35.87
6.70	19.701	0.63651	0.52230	39.86
6.71	19.672	0.63841	0.52343	40.06

$\therefore b_{min} = 6.71$ in. ←

Problem 11.9-34 A square wood column with side dimensions b (see figure) is constructed of a structural grade of southern pine for which $F_c = 10.5$ MPa and $E = 12$ GPa. An axial force $P = 200$ kN acts on the column.

(a) If the dimension $b = 150$ mm, what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 4.0$ m, what is the minimum required dimension b_{min}?

Solution 11.9-34 **Wood column (square cross section)**

$F_c = 10.5$ MPa $\quad E = 12$ GPa $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 200$ kN

(a) MAXIMUM LENGTH L_{max} FOR $b = d = 150$ mm

From Eq. (11-92): $C_P = \dfrac{P}{F_c b^2} = 0.84656$

From Eq. (11-95):

$$C_P = 0.84656 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 1.7807$

From Eq. (11-94): $\dfrac{L}{d} = \sqrt{\dfrac{K_{cE}E}{\phi F_c}} = 13.876$

$\therefore L_{max} = 13.876\, d = (13.876)(150 \text{ mm})$
$= 2.08$ m ←

(b) MINIMUM DIMENSION b_{min} FOR $L = 4.0$ M

Trial and error: $\dfrac{L}{d} = \dfrac{L}{b} \quad \phi = \dfrac{K_{cE}E}{F_c(L/d)^2}$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \qquad P = F_c C_P b^2$$

Given load: $P = 200$ kN

Trial b (mm)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (kN)
180	22.22	0.69429	0.55547	189.0
182	21.98	0.70980	0.56394	196.1
183	21.86	0.71762	0.56814	199.8
184	21.74	0.72549	0.57231	203.5

$\therefore b_{min} = 184$ mm ←

Problem 11.9-35 A square wood column with side dimensions b (see figure) is constructed of a structural grade of spruce for which $F_c = 900$ psi and $E = 1{,}500{,}000$ psi. An axial force $P = 8.0$ k acts on the column.

(a) If the dimension $b = 3.5$ in., what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 10$ ft, what is the minimum required dimension b_{min}?

Solution 11.9-35 **Wood column (square cross section)**

$F_c = 900$ psi $\quad E = 1{,}500{,}000$ psi $\quad c = 0.8$

$K_{cE} = 0.3 \quad P = 8.0$ k

(a) MAXIMUM LENGTH L_{max} FOR $b = d = 3.5$ IN.

From Eq. (11-92): $C_P = \dfrac{P}{F_c b^2} = 0.72562$

From Eq. (11-95):

$$C_P = 0.72562 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 1.1094$

From Eq. (11-94): $\dfrac{L}{d} = \sqrt{\dfrac{K_{cE}E}{\phi F_c}} = 21.23$

$\therefore L_{max} = 21.23\, d = (21.23)(3.5 \text{ in.}) = 74.3$ in. ←

(b) MINIMUM DIMENSION b_{min} FOR $L = 10$ FT

Trial and error. $\frac{L}{d} = \frac{L}{b}$ $\phi = \frac{K_{cE}E}{F_c(L/d)^2}$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \qquad P = F_c C_P b^2$$

Given load: $P = 8000$ lb

$\therefore b_{min} = 4.20$ in. ⟵

Trial b (in.)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (lb)
4.00	30.00	0.55556	0.47145	6789
4.20	28.57	0.61250	0.50775	8061
4.19	28.64	0.60959	0.50596	7994

Problem 11.9-36 A square wood column with side dimensions b (see figure) is constructed of a structural grade of eastern white pine for which $F_c = 8.0$ MPa and $E = 8.5$ GPa. An axial force $P = 100$ kN acts on the column.

(a) If the dimension $b = 120$ mm, what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 4.0$ m, what is the minimum required dimension b_{min}?

Solution 11.9-36 **Wood column (square cross section)**

$F_c = 8.0$ MPa $E = 8.5$ GPa $c = 0.8$

$K_{cE} = 0.3$ $P = 100$ kN

(a) MAXIMUM LENGTH L_{max} FOR $b = d = 120$ mm

From Eq. (11-92): $C_P = \frac{P}{F_c b^2} = 0.86806$

From Eq. (11-95):

$$C_P = 0.86806 = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 2.0102$

From Eq. (11-94): $\frac{L}{d} = \sqrt{\frac{K_{cE}E}{\phi F_c}} = 12.592$

$\therefore L_{max} = 12.592\, d = (12.592)(120 \text{ mm})$

$= 1.51$ m ⟵

(b) MINIMUM DIMENSION b_{min} FOR $L = 4.0$ m

Trial and error. $\frac{L}{d} = \frac{L}{b}$ $\phi = \frac{K_{cE}E}{F_c(L/d)^2}$

$$C_P = \frac{1+\phi}{1.6} - \sqrt{\left[\frac{1+\phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \qquad P = F_c C_P b^2$$

Given load: $P = 100$ kN

Trial b (mm)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (kN)
160	25.00	0.51000	0.44060	90.23
164	24.39	0.53582	0.45828	98.61
165	24.24	0.54237	0.46269	100.77

$\therefore b_{min} = 165$ mm ⟵